# Further Analyses Related to the Application of Statistical Catch-at-age Analysis to Data for Southern Hemisphere Minke Whales 

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#### Abstract

Results of configurations of the statistical catch-at-age analysis developed for the Southern Hemisphere minke whales show that the lack of a positive definite Hessian matrix in recent applications of this analysis method was due to the penalty imposed to prevent the predicted number of calves exceeding the corresponding number of mature females. The problem can be overcome (and a positive definite Hessian matrix achieved) by assuming that the age-specific pattern of natural mortality is independent of stock (but that the average value for natural mortality is stock-specific). Diagnostic plots based on observed and model-predicted mean ages-at-length by year, sex and fleet show that the model is able to mimic changes over time in size-at-age for Southern Hemisphere minke whales. All but one of the analyses of this paper (including sensitivity tests in which ex-USSR length data are omitted and in which the age-reading error matrix is changed) indicate that the abundance of the "W" and "E" stocks increased from the 1930s until the early 1970s and have declined thereafter. The exception occurs when Zenitani (rather than Lockyer) is assumed to be an unbiased age-reader. This exception disappears when the (Japanese) length-frequency data for 1972 are ignored.


## KEYWORDS: CATCH-AT-AGE, MINKE WHALE, SOUTHERN HEMISPSHERE

## INTRODUCTION

Two variants of catch-at-age analysis have been applied to data for the Antarctic minke whales in recent years. Mori and colleagues (e.g. Mori et al., 2007; Mori and Butterworth, 2008) have applied ADAPT-VPA while Punt and Polacheck (e.g. Punt and Polacheck (2005, $2006,2007,2008)$ ) have applied statistical catch-at-age analysis. With a few exceptions, both of these techniques concluded that the population size of Antarctic minke whales increased from 1930 to the early 1970s, and declined thereafter.

Punt (2010) conducted sensitivity tests to explore impact of allowing for ageing error using estimates of ageing error derived by Kitakado and Punt (2010). However, the analyses based on these estimates did not lead to a positive definite Hessian matrix and Punt (2010) recommended that the reasons for this need to be understood and the results confirmed before final conclusions can be drawn.

The paper reports on the reasons for the lack of a positive definitive Hessian matrix, and the impact of alternative assumptions to the assumption made by Punt (2010) that the ageestimates made by Lockyer are unbiased. As recommended by Subcommittee on In-depth Assessment (IWC, 2011), this paper also investigates (a) whether statistical catch-at-age model together with its various estimated selectivity functions can account satisfactorily for the different length-at-age distributions for younger animals in the commercial and JARPA catches, and (b) the impact of possible misreporting of the length distribution by the USSR commercial fleet. IWC (2011) also recommended conducting additional sensitivity tests using updated JARPA data. However, there are no new JARPA data (L.A. Pastene, pers. commn). The data and basic method of analysis is therefore identical to that used by Punt and

Polacheck (2008). This analysis considers the years 1930/31 to 2003/04 (although catch data are only available from 1953/54 and catch-at-age data are only available from 1971/72).

## REASONS FOR THE LACK OF A POSITIVE-DEFINITIVE HESSIAN MATRIX

Earlier versions of the statistical catch-at-age analysis (e.g. Punt and Polacheck, 2008) led to positive definite Hessian matrices (and hence estimates of variance for model outputs). However, the most recent applications of the statistical catch-at-age analysis have not. A variety of model formulations (results not shown) were examined and the key feature which determined whether the Hessian matrix was positive definite or not was the penalty imposed on the ratio of calves to mature females.

The time-trajectory of the number of calves per mature female for the two stocks ("W" and "E") exceeds 1 for stock " $E$ " when no penalties are imposed on this ratio (model "original"; Figure 1, right panel). This is clearly unrealistic and a large penalty was consequently added to the objective function when this occurred. While this ensured that the numbers of calves per mature female did not exceed one, it led to a non-differentiable objective function. The reason for the unrealistic number of calves per mature female is that the rate of natural mortality for calves is estimated to be $0.184 \mathrm{yr}^{-1}$ for the "E" stock (compared to $0.070 \mathrm{yr}^{-1}$ for the "W" stock) [Figure 2] when no penalties are imposed as this leads to slightly better fits to the data.

A number of alternative model configurations were developed based on different types of penalties, all of which aimed to ensure that the ratio of calves per mature female would not exceed 1. Unfortunately, the Hessian matrix was either not positive definitive for these model configurations or the penalty was insufficient to ensure that that the number of calves was always less than the number of mature females. However, a model in which the change in natural mortality had the same pattern for the "E" and "W" stocks (but the average rate of natural mortality differed among stocks) was able to impose the desired penalty, and led to a Hessian matrix that was positive definite. For this model configuration, the relationship between natural mortality and age was taken to be piecewise linear, but with parameters $\delta$ and $\gamma$ which were independent of stock:

$$
M_{a}^{s}= \begin{cases}\delta M^{s} & \text { if } a \leq a_{1}  \tag{1}\\ M^{s}\left[\delta+(1-\delta) \frac{\left(a-a_{1}\right)}{\left(a_{2}-a_{1}\right)}\right] & \text { if } a_{1}<a<a_{2} \\ M^{s} & \text { if } a_{2} \leq a \leq a_{3} \\ M^{s}\left[1+(\gamma-1) \frac{\left(a-a_{3}\right)}{\left(a_{4}-a_{3}\right)}\right] & \text { if } a_{3}<a<a_{4} \\ \gamma M^{s} & \text { if } a \geq a_{4}\end{cases}
$$

where $\delta M^{s}$ is the rate of natural mortality for animals of stock $s$ aged $a_{1}(3)$ and younger,
$M^{s} \quad$ is the rate of natural mortality for animals of stock $s$ aged between $a_{2}$ and $a_{3}$ (10 and 30 respectively) and
$\gamma M^{s}$ is the rate of natural mortality for animals aged $a_{4}$ (35) and older.
The "original" model treated $\gamma$ and $\delta$ as stock-specific parameters and hence estimated two additional parameters compared to the model based on Equation 1. The difference in the lowest value of the objective function between the (unrealistic) "original" model and the model in which $\gamma$ and $\delta$ are independent of stock ("original alternative") is 8.05 which, while
supporting the "original" model over the "original alternative" model, is hardly an enormous difference.

Conducting an analysis in which $\gamma$ and $\delta$ are independent of stock, but using the ageing error model of Kitakado and Punt (2010) [analysis "with new ageing error"; henceforth "base case"] (unlike the semi-arbitrary ageing error matrix on which the "original" model configuration was based) leads to time-trajectories of total (1+) abundance and recruitment which suggest that the abundance of stocks "W" and "E" increased between 1930 and the early 1970s and has declined thereafter (Fig. 2). The rate of increase is, however, lower for the W stock when the new ageing-error matrix is used.

An original reason for investigating models for which the Hessian matrix is positive definitive was that Punt (2010) was unable to conclude that allowing for a more data-based ageing error matrix when conducting assessments for the "W" and "E" stocks did not impact the earlier conclusions regarding trends in abundance. Figure 3 therefore compares the "base case" analysis of this paper with the corresponding analysis by Punt (2010). The results for stock "W" are essentially identical between those obtained by Punt (2010) and in this paper. In contrast, historical abundance of stock "E" is impacted fairly markedly by the choice of ageing error matrix [this is not the case for recent years because abundance in these years is constrained by the IDCR and JARPA estimates of abundance]. The reason for the differences in scale between the two base-case analysis can be attributed to the estimates of natural mortality (primarily for the youngest ages; Figure 3). Nevertheless, the results in Figure 3 suggest that allowing for ageing error in the manner considered by Punt (2010) does not impact the conclusions of previous analyses based on the statistical catch-at-age analysis of an increase in minke whale abundance before the early 1970s.

## ABILITY TO MIMIC THE SCIENTIFIC AND JARPA LENGTH-AT-AGE DATA

One of the original reasons for further investigating ageing error was the observation by Punt and Polacheck (2005) that there were substantial differences between the estimated growth curves depending on whether they were estimated internally within the model based on the length and age data collected from the commercial catches or externally using JARPA data. Polacheck and Punt (2006) explored alternative growth models with time-varying growth rates which also allow for changes over time in vulnerability to assess whether these factors can account for the inconsistency, and suggested that a combination of temporal changes in growth rates and length-based vulnerability can explain a large portion of the apparent inconsistency, with changes in growth being the dominant factor. Polacheck and Punt (2006) assessed lack of fit using observed and model-predicted length-frequencies by age. However, the agelength data are fitted as conditional age-length length data when applying the statistical catch-at-age analysis method [e.g. Punt et al., 2006] (rather than as age-compositions) which necessitates the use of appropriate diagnostics statistics. Such statistics have been developed in the context of the application of statistical catch-at-age methods for fisheries assessments (e.g. the r4ss package; Taylor et al., 2009).

Figures 4 and 5 plot the observed mean age for each length-class (points), along with the corresponding model estimate (lines) for each combination of sex, fleet and year. This differs from the standard diagnostics for age-at-length data because it accounts for the fact that the model fits the age-composition data conditional on length-class. The results in Figures 4 and 5 are only based on Japanese data because there are no USSR ageing data. While there are some obvious outliers (caused by small sample sizes, primarily for the largest and smallest animals in the catch), the model is generally able to mimic the data very well. A noteworthy exception to this is the fairly poor fits to the data for males in Area III-E in 1998, 2000, and 2002 (Figure 5a). Most importantly perhaps, there is no evidence for a change in the signs of the residuals between the commercial and JARPA periods, which suggests that the model is
able to adequately fit the data (albeit by allowing for both time-varying fishery [but not JARPA] selectivity and time-varying growth).

## SENSITIVITY TO OMITTING THE USSR LENGTH DATA

IWC (2011) recommended assessing the impact of possible misreporting of the length distribution by the USSR commercial fleet on the results of the statistical catch-at-age analysis. Figure 6 therefore shows results for a variant of the base-case analysis in which it is assumed that the selectivity patterns for the USSR catches are the same as those for the Japanese fleets and ignores the length-composition data for the USSR fleets when fitting the model. As noted above, all of the age data were collected by Japan so there was no need to ignore any USSR age data. The USSR catches are relatively small compared the estimates of abundance when these catches were taken so any misreporting of the numbers caught would be highly unlikely to impact the inferences drawn.

Figure 6 shows that the while the estimates of recruitment and natural mortality for the youngest animals are sensitive to dropping the USSR length-frequency data, the overall conclusion of a population which increased from 1930 until the late 1970s remains robust. The somewhat higher natural mortality on younger animals is, in fact, arguably more realistic that the estimates arising from the base-case analysis.

## SENSITIVITY TO ALTERNATIVE AGEING ERROR MATRICES

IWC (2011) recommended that alternative assumptions in regard to Lockyer's ageing bias be explored. Figure 7 therefore reports results for five scenarios.

- The base-case analysis (Lockyer is an unbiased reader).
- Lockyer provides estimates with are negatively biased by $10 \%$ (Lockyer A).
- Lockyer provides estimates with are positively biased by $10 \%$ (Lockyer B).
- Zenitani provides estimates which are unbiased (the bias for Lockyer is estimated).
- All age-readers are unbiased.

The results for stock E are robust to how the age-reading error matrices are constructed. However, assuming that Zenitani is an unbiased age-reader leads to higher estimated rates of natural mortality, and time-trajectories of $1+$ population size and recruitment for the W stock which are essentially constant before 1970. Further exploration of model configurations suggests that the primary reason for the difference in results is the length-frequency data for 1972 (the first year for which such data are available) (Figure 8). Length-frequency data are available for Japan in Areas III-E and Area IV for 1972 and these length-frequencies tend to have very few small animals.

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Figure 1. Time-trajectories for the number of calves divided by the number of mature females for (a) an analysis ("original") which matches the base-case analysis of previous papers applying the statistical catch-at-age analysis method, (b) an analysis ("original alternative") which uses the same data as the "original" analysis, but assumes that the change in natural mortality by age is the same for each stock, and (c) a variant of the "original alternative" analysis in which the age-reading error matrix of Kitakado and Punt (2010) is used.


Figure 2. Time-trajectories of total (1+) population size, age-0 recruitment and natural mortality for the three analyses in Figure 1.


Figure 3. As for Figure 2, except that results are shown for the "Base case" analysis of this paper and the base-case analysis of Punt (2010).


Figure 4 a . Observed (dots) and base-case model-predicted (lines) mean ages versus length (ft) for females in Area III-E.


Figure 4b. As for Figure 4a, except that the results pertain to Area IV.


Figure 4c. As for Figure 4a, except that the results pertain to Area V-W.


Figure 4d. As for Figure 4a, except that the results pertain to Area V-E.


Figure 4e. As for Figure 4a, except that the results pertain to Area IV-W.


Figure 5a. Observed (dots) and base-case model-predicted (lines) mean ages versus length (ft) for females in Area III-E.


Figure 5b. As for Figure 5a, except that the results pertain to Area IV.


Figure 5c. As for Figure 5a, except that the results pertain to Area V-W.


Figure 5d. As for Figure 5a, except that the results pertain to Area V-E.


Figure 5 e . As for Figure 5a, except that the results pertain to Area IV-W.


Figure 6. Time-trajectories of total (1+) population size, age-0 recruitment and natural mortality for the base-case analysis and variants therefore which ignores the USSR lengthcomposition data and assumes instead that the selectivity patterns for the USSR fleets were the same as those of the Japanese fleets.


Figure 7. Time-trajectories of total (1+) population size, age-0 recruitment and natural mortality for the base-case analysis and variants therefore that vary the basis for the agereading error matrices.


Figure 8. Time-trajectories of total ( $1+$ ) population size, age-0 recruitment and natural mortality for the base-case analysis, the variant in which Zenitani is assumed to be unbiased and a variant thereof in which the length-frequency data for 1972 are ignored.

