A note on the long-term behaviour of the RMP Catch Limit Algorithm

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Results presented in SC/58/RMP 7 and SC/59/RMP 4 suggest that the CLA median final population level in standard scenarios is noticeably higher after 300 years than after 100 years. A retuning of the CLA to achieve a specified median final depletion, such as 0.72*K*, after 300 years would involve a lower (less conservative) tuning than one designed to hit the same target level after 100 years.

It is not clear how general this behaviour is. The CLA was tested on the following scenario:

Population model: age structured, sex-equal (all parameters identical for both sexes)
Age at first calving: $T_{m} =$ 8 years
Age at first capture: $T_{r} =$ 8 years
Natural mortality rate (age 1+): $M = 0.07$
Natural mortality rate (age 0): from balance equation at $P_{\text{mat}} = K_{\text{mat}}$.
Density-dependent *per capita* recruitment function:

$$r(P_{\text{mat}}) = r(K_{\text{mat}})\ (1 + A)^{1-(P_{\text{mat}}/K_{\text{mat}})^z}$$

where:
Resilience: $A = 0.192$
Density-dependent exponent: $z = 2.6$
A, Z chosen to give $MSYR_{\text{mat}} = 0.01$, $MSYL = 0.6K_{\text{mat}}$

Survey frequency: every 5 years
Survey CV: 0.25 observed + 0.25 additional variance = 0.353 total.
Initial depletion: 1.0

This scenario is virtually identical to the standard D1 scenario developed by C. Allison except that the Pella-Tomlinson recruitment curve has been replaced by a Ricker curve.

Fig. 1 shows the trajectory of the 5$^{\text{th}}$, 50$^{\text{th}}$ and 95$^{\text{th}}$ percentiles of median mature depletion, based on 300 replicates, using the IWC 0.72 tuning of the CLA (tuning parameter 0.4080). A high-precision computer programme was used to perform the calculations, tuned to calculate the catch limit corresponding to the target percentile (0.4080) with a tolerance of 0.0001 in the percentile.

The median depletion is approximately stable after 50 years and shows no tendency to increase after 100 years. The median depletion is 0.689 after 100 years and 0.679 after 300 years.

It is not clear what is causing the difference from the results of SC/58/RMP 7and SC/59/RMP 4. It would seem unlikely to be due to the change from a P-T to a Ricker stock recruitment curve because Punt (Email of 4 Oct. 2006, appended) found this change made essentially no difference to the behaviour of the CLA.

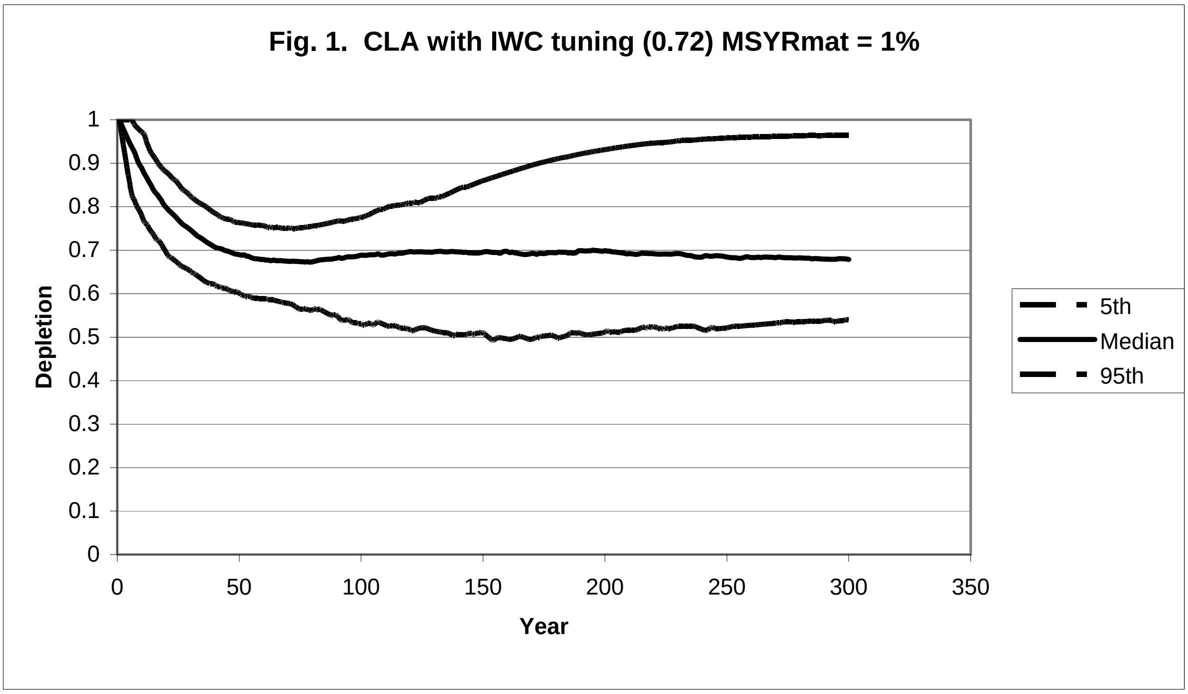
Fig. 1. CLA with IWC tuning (0.72) MSYRmat = 1%
Depletion
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
50
100
150
200
250
300
350
Year
5th
Median
95th

Appendix 1

A note on functional forms for the density-dependence in stock-recruitment in the operating model of the single-stock control programme for simulations of the CLA, with reference to behaviour for *P* > *K*

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The following function determines the extent of density-dependence (and hence the number of births) in the single-stock control program:

$$B_t = \max\{0, \tilde{N}_t\ f_0(1+A_t(1-[\tilde{P}_t/\tilde{K}_t]^{z_t}))\} \tag{1}$$

where B_t is the number of births at the start of year *t*,

$\tilde{N}_t$ is the number of females that have reached the age-at-first-parturition at the start of year *t*,

f_0 is fecundity (number of female calves per female that has reached the age-at-first-parturition) at pre-exploitation equilibrium,

A_t is the resilience parameter during year *t*,

z_t is the degree of compensation during year *t* (*A* and *z* depend on time to implement the trials in which *MSYR* is time-dependent),

$\tilde{P}_t$ is the size of the density-dependent component of the population (recruited, total (1+), or mature) at the start year *t*, and

$\tilde{K}_t$ is the carrying capacity of the density-dependent component of the population at the start of year *t*.

Equation 1 has the (seemingly) undesirable property that the number of births is zero if $\tilde{P}_t > \tilde{K}_t\left[1+1/A_t\right]^{1/z_t}$. It is therefore desirable to find a replacement for Equation 1 that reduces the number of births if the population size exceeds (current) carrying capacity, but does not lead to zero births. A functional form that has this property is the Ricker model, i.e.:

$$B_t = \tilde{N}_t\ f_0 \exp[\alpha_t(1-\tilde{P}_t/\tilde{K}_t)] \tag{2}$$

where α_t is the parameter that determines the extent of density-dependence.

Equation 2 satisfies the requirements that: a) fecundity equals f_0 when $\tilde{P}_t = \tilde{K}_t$, and b) the number of births never equals zero. However, Equation 2 has only one density-dependence-related parameter (α) so it is not possible to simultaneously specify *MSYR* and *MSYL* if density-dependence is governed by the Ricker model.

There are two possible ways to proceed at this point. Either, the Ricker model can be generalized by adding an extra parameter so that it becomes possible to specify *MSYR* and *MSYL* simultaneously, or Equation 1 can be modified so that the number of births does not equal 0 for any population size. The most natural generalization of the Ricker model would be:

$$B_t = \tilde{N}_t \; f_0 \exp[\alpha_t(1-(\tilde{P}_t/\tilde{K}_t)^{\beta_t})] \tag{3}$$

while Equation 1 could be modified to:

$$B_t = \begin{cases} N_t \; f_0(1+A_t(1-[\tilde{P}_t/\tilde{K}_t]^{z_t})) & \text{if } \tilde{P}_t/\tilde{K}_t \le 1 \\ N_t \, f_0 \exp(A_t(1-[\tilde{P}_t/\tilde{K}_t]^{z_t})) & \text{otherwise} \end{cases} \tag{4}$$

Equation 4 has the same properties as Equation 3, but requires much less coding to implement in the common control program (and there is no need to re-derive the relationship between *MSYR* / *MSYL* and the parameters of the density-dependence function). The remaining analyses of this note are therefore based on Equation 4.

Figure 1 compares the application of Equations 1 and 4 (solid and dashed lines respectively) for four levels for the resilience parameter (the value of z is set to 2.39 for all of the plots in Figure 1). Table 1 compares a subset of the standard performance statistics for 100-year simulations of the 0.6 and 0.72 tunings of the *CLA* when Equations 1 and 4 are used to model the impact of density-dependence. Results are shown for the four base-case trials and the trials in which K halves over the 100-year projection period and the initial population size is 0.99K. Results are shown for trials in which the population size is initially 0.99K (rather than 0.3K) because Equations 1 and 4 only lead to different outcomes when $\tilde{P}_t/\tilde{K}_t$ >1. Results (not shown in Table 1) confirm that the conclusion of negligible differences between including Equation 4 rather than Equation 1 in the common control program is robust to whether *MSYR* (in the operating model) is defined in terms of mature or 1+ components of the population.

The performance statistics for the two ways of modelling density-dependence differ only negligibly. This suggests that it is not particularly important to select between Equations 1 and 4 (although Equation 4 is arguably more aesthetically pleasing). The reasons for the lack of difference between the results for the trials based on Equation 1 and those based on Equation 4 are: a) even though K halves over the 100-year projection period, the change in K in any one year is fairly small so $\tilde{P}_t/\tilde{K}_t$ never gets sufficiently large that there are major differences between Equations 1 and 4 (such difference might arise had K be reduced to 5% of its initial value over the 100-year period), and b) scaling the population trajectories by the population trajectory when catch is zero tends to reduce much of the impact of the difference between Equations 1 and 4.

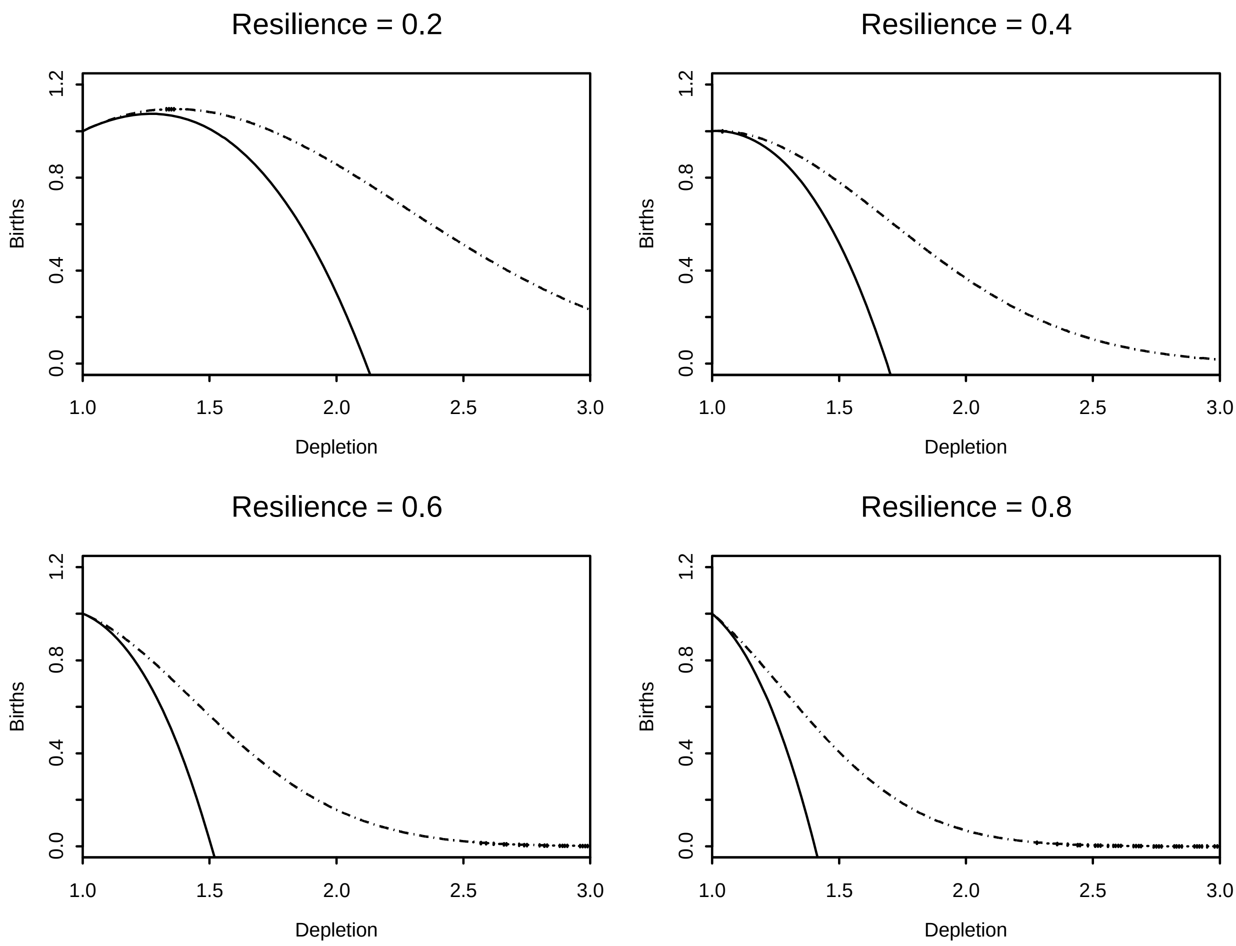

Figure 1. Births (relative to those at *K*) versus the depletion of the mature component of the population. Results are shown for Equation 1 (solid line) and Equation 4 (dashed line).

Table 1. Performance statistics for the 0.72 and 0.60 tunings of the *CLA*. The trials in which density-dependence is governed by Equations 1 and 4 are indicated respectively by an "A" or "B" at the end of the trial abbreviation. The performance statistics for the trials in which *K* halves (T12B) have been calculated after scaling the population and catch trajectories by the population size trajectory for the "no catch" scenario.

	Total catch				Final population (mature)			Final population (1+)			Low population (mature)		
	Median	5%	96%	Mean	Median	5%	96%	Median	5%	96%	5%	10%	25%
0.72 tuning													
T1-D1A	0.876	0.745	1.049	0.883	0.723	0.609	0.805	0.746	0.637	0.825	0.602	0.623	0.655
T1-D1B	0.876	0.745	1.049	0.883	0.723	0.609	0.805	0.746	0.637	0.825	0.602	0.623	0.655
T1-D4A	1.146	0.930	1.401	1.153	0.913	0.842	0.973	0.959	0.914	0.989	0.792	0.809	0.832
T1-D4B	1.146	0.930	1.401	1.153	0.913	0.842	0.973	0.959	0.914	0.989	0.792	0.809	0.832
T1-R1A	0.101	0.035	0.200	0.107	0.615	0.547	0.658	0.637	0.569	0.680	0.300	0.300	0.300
T1-R1B	0.101	0.035	0.200	0.107	0.615	0.547	0.658	0.637	0.569	0.680	0.300	0.300	0.300
T1-R4A	0.478	0.257	0.793	0.499	0.950	0.895	0.985	0.978	0.948	0.994	0.300	0.300	0.300
T1-R4B	0.478	0.257	0.793	0.499	0.950	0.895	0.985	0.978	0.948	0.994	0.300	0.300	0.300
T12B-D1A	0.788	0.679	0.922	0.800	0.554	0.480	0.598	0.548	0.487	0.583	0.480	0.511	0.535
T12B-D1B	0.788	0.679	0.922	0.800	0.554	0.480	0.599	0.549	0.487	0.584	0.480	0.511	0.535
T12B-D4A	0.855	0.706	1.016	0.858	0.533	0.499	0.544	0.514	0.501	0.518	0.499	0.513	0.523
T12B-D4B	0.855	0.708	1.016	0.858	0.535	0.500	0.548	0.517	0.501	0.521	0.500	0.514	0.524
0.6 tuning													
T1-D1A	1.112	0.934	1.303	1.112	0.603	0.466	0.720	0.628	0.492	0.743	0.455	0.482	0.526
T1-D1B	1.112	0.934	1.303	1.112	0.603	0.466	0.720	0.628	0.492	0.743	0.455	0.482	0.526
T1-D4A	1.624	1.327	1.925	1.621	0.847	0.766	0.940	0.917	0.859	0.973	0.716	0.735	0.762
T1-D4B	1.624	1.327	1.925	1.621	0.847	0.766	0.940	0.917	0.859	0.973	0.716	0.735	0.762
T1-R1A	0.225	0.135	0.350	0.231	0.515	0.424	0.582	0.538	0.444	0.604	0.300	0.300	0.300
T1-R1B	0.225	0.135	0.350	0.231	0.515	0.424	0.582	0.538	0.444	0.604	0.300	0.300	0.300
T1-R4A	0.894	0.584	1.189	0.895	0.891	0.834	0.955	0.945	0.909	0.982	0.300	0.300	0.300
T1-R4B	0.894	0.584	1.189	0.895	0.891	0.834	0.955	0.945	0.909	0.982	0.300	0.300	0.300
T12B-D1A	1.022	0.868	1.202	1.024	0.493	0.384	0.567	0.497	0.399	0.559	0.384	0.427	0.459
T12B-D1B	1.022	0.868	1.202	1.024	0.493	0.384	0.567	0.497	0.399	0.559	0.384	0.427	0.459
T12B-D4A	1.216	1.002	1.460	1.223	0.519	0.457	0.543	0.511	0.482	0.518	0.457	0.479	0.498
T12B-D4B	1.216	1.002	1.460	1.223	0.520	0.457	0.547	0.511	0.482	0.522	0.457	0.479	0.499