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Constructing a Posterior Distribution for the Rate of Increase of Whale Stocks at Low Population Size

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ABSTRACT
An approach for conducting a meta-analysis of rates of increase at low population size, r_0, for whales stocks based on the Bayesian paradigm is outlined. This approach is then applied for illustrative purposes to the estimates of r_0 assembled by the November 2007 MSYR workshop.
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KEYWORDS: META-ANALYSIS; MSYR

INTRODUCTION

The Scientific Committee of the International Whaling Commission is conducting a review of the range of MSYR values to include in simulation trials when selecting among variants of the Revised Management Procedure. A table of estimates of MSYR and rates of increase at low population size, r_0, was developed by IWC (2008) as a part of this review. One of the recommendations of IWC (2008) was that a meta-analysis of the estimates of r_0 should be conducted, ideally treating species as a covariate.

There are several methods for conducting meta-analyses. Here a Bayesian approach to meta-analysis is applied, with the objective of characterizing the distribution for r_0 for a randomly selected (and unknown) stock of baleen whale.

MATERIALS AND METHODS

The data

Table 1 lists the estimates of r_0 and $MSYR_{1+}$ from IWC (2008)[1], excluding the values that were rejected. There is more than one estimate of r_0 for some stocks; Table 1 provides the estimates that were considered to be of highest reliability. Although Table 1 lists all of the estimates reported by IWC (2008), the illustrative application is restricted to data for stocks for which estimates of r_0 are available, excluding that for eastern North Pacific stock of gray whales which has recovered to a substantial extent and hence for which the increase rate is not applicable to stocks at low population size.

The Bayesian meta-analysis

The basic structure of the data and how they are analysed is outlined in Fig. 1. The data (the estimates of r_0 and their observation error standard errors) are assumed to be measures of the true values for r_0. The true values for r_0 are, in turn, assumed to be drawn (at random) from a normal hyper-distribution, parameterized in terms of a mean μ, and a standard deviation σ. The values for the parameters of the hyper-distribution and the

[1] Some of these values will be updated for the February 2009 workshop, to which this document will be presented.

values of r_0 for each stock are estimated using Bayesian methods. For the purposes of this paper a wide uniform prior is placed on μ and the prior for σ is assumed to be U(0, 100].

The posterior is summarized using samples selected at random from the posterior distribution using the Gibbs Sampler as implemented in WinBUGS (Spiegelhalter *et al*. 2002). The posterior distributions for μ and σ were confirmed by independently implementing the model using the random effects module of AD Model Builder and drawing samples from a Bayesian posterior distribution with the same priors for μ and σ as were used in the WinBUGS analyses. The posterior distribution for the rate of increase for a "unknown" stock was constructed by sampling values for μ and σ from the joint posterior distribution (to allow for uncertainty about the values for the hyper-parameters) and then generating a rate of increase from $N(\mu, \sigma^2)$ many times.

RESULTS AND DISCUSSION

Preliminary application

The posterior distribution is summarized by 1,000 draws from the posterior distribution constructed by running the Gibbs sampler for 500,000 cycles (discarding the first 100,000 cycles as a burn-in period). Figure 2 shows the posterior distributions for μ and σ, and Figure 3 that for the rate of increase for an 'unknown' stock. As expected, the width of the latter distribution is greater than that for μ.

Figure 4 explores the extent to which the analyses update the original point estimates of r_0 by stock, taking account of the information provided on the rate of increase from the population mean. The extent to which the estimates of r_0 in Table 1 are "shrunk" to μ depends on the extent of observation error. For example, the posterior mean for the rate of increase for East Africa humpbacks is 9.7% although the point estimate is 12.1%. Similarly, the standard errors for the estimates of stock-specific r_0 for those stocks for which the estimates of r_0 are very imprecise are shrunk towards the posterior mean for σ.

Final comments

There are some major assumptions of the meta-analysis which need to be taken into account when the results are interpreted. Four of these are:

1) All stocks are interchangeable, i.e. the stocks for which estimates of r_0 are available are a random selection of possible stocks (from the population of stocks) and there is no selection for stocks based, for example, on their rates of increase.
2) There are no covariates which can separate the stocks, in terms of their rates of increase.
3) The standard errors of the observation errors (by stock) are known without error.
4) The rates of increase relate to stocks at a small fraction of their carrying capacities so that the impacts of density-dependence can safely be assumed to be negligible. This may not be the case for some of the stocks in Table 1 (e.g. the Bering-Chukchi-Beaufort Seas stock of bowhead whales).

This analysis takes some ten minutes to run. It can readily be repeated during the February 2009 workshop, both for amended input data and for subsets of the data restricted to a certain group of species.

REFERENCES

IWC. 2008. Report of the MSYR Workshop. Document SC/50/Rep5 presented to the IWC Scientific Committee, June 2008. (unpublished). 16pp.

Spiegelhalter, D.J., Best, N.G., Carlin, B.P., and Van der Linde, A. 2002. Bayesian measures of model complexity and fit. J. Royal Stat. Soc. Series B, 64: 583-616.

Table 1
Estimates of r_0 (rate of increase at low population size) and $MSYR_{1+}$ (Table 1 of IWC 2008). Only the data for the stocks for which estimates of r_0 are available are used in the illustrative application. Standard deviations are computed from reported confidence intervals where standard deviations are not reported in IWC (2008).

Stock	**$MSYR_{1+}$**	**r_0 (%)**	**SE**	**Reliability**
Blue				
Central North Atlantic		9.0 (2.0-17.0)	3.83[a, b]	High
Southern Hemisphere		8.2 (3.8 – 12.5)	2.22[a,c]	High
Fin				
East Greenland Iceland	1.7 (1.0-2.9)			Medium
North Norway		5 (-13, 26)	9.95[a]	Unreliable
Eastern North Pacific		4.8 (-1.6, 11.1)	3.24[a]	Unreliable
Southern Hemisphere		10.2 (4.8-15.6)	2.76[a]	???
Minke				
Indian	5.5 (SE 0.5)			Low
Pacific	3.6 (SE 2.6)			Low
Northeast Atlantic	1.9 (<0.1, 3.84)			Medium
Humpback				
Northwest Atlantic		3.1	0.5	High
Brazil		7.4 (0.6, 14.5)	3.55[a]	Medium
East Africa		12.1 (7.1, 17.1)	2.55[a]	Medium
Western Australia		10.1 (0.9, 19.3)	4.69[a]	???
Eastern Australia		10.6 (10.1, 11.1)	0.26[a]	High
Gray				
Western		2.9 1.9-2.0	0.54[d]	High
Eastern	7.0 (4.8-9.2)	1.86	0.32[e]	High
Bowhead				
Bering-Chukchi-Beaufort	3.3 (1.9-4.8)	3.4 (1.7-5.0)	0.84[d]	High
Southern Right				
SE Atlantic		7.3 (6.6-7.9)	0.33[a]	High
SW Atlantic		6.8 (5.8-7.8)	0.51[a]	High
SW Pacific		8.3 (5.1-11.4)	1.61[a]	High

a – computed from the 95% confidence interval by dividing by 3.92
b – used in preference to estimates based on SPUE
c – used in preference to the integrated trend analysis (as that uses SPUE)
d – computed from the 90% confidence interval by dividing by 3.28
e – not used; stock is well recovered.

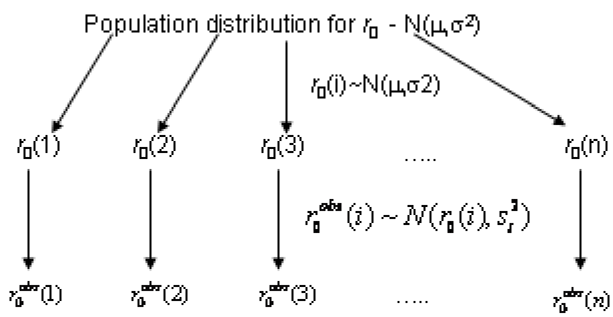

Figure 1. Outline of the meta-analysis. μ and σ are, respectively, the population mean and population standard deviation for the rate of increase, $r_0(i)$ is the "true" rate of increase at low population size for stock *i*, $r_0^{obs}(i)$ is the observed rate of increase for stock *i*, and s_i is the observation error standard error for stock *i* (assumed to be known without error).

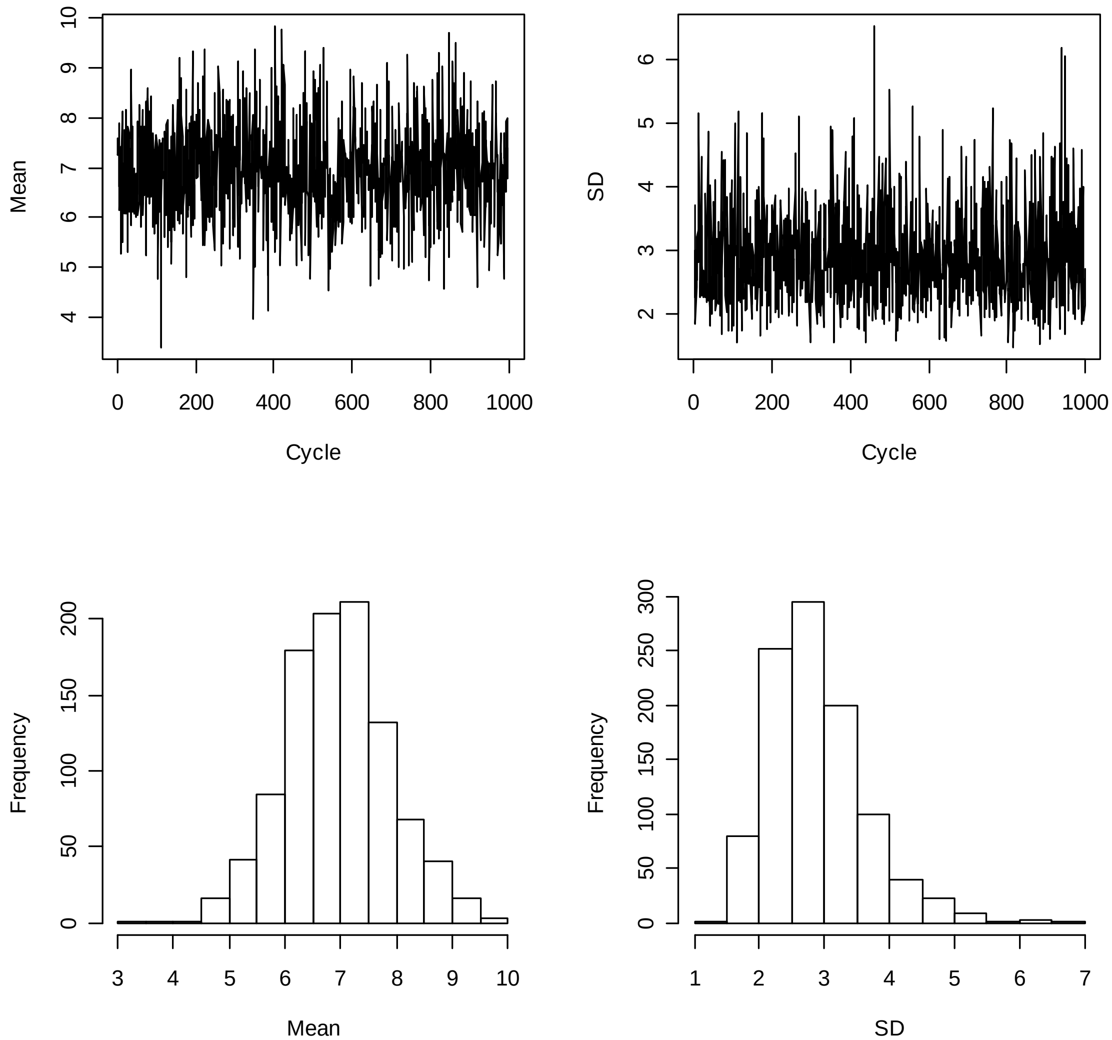

Figure 2. Posterior distribution for the population mean values for the rate of increase (expressed as percentage) at low population size, r_0, and the between-population standard deviation for the rate of increase. The upper plots are traces and provide no indication of lack of convergence. The means and standard deviation of the hyper-distributions for the population mean and standard deviation are respectively 6.93/0.94 and 2.88/0.74.

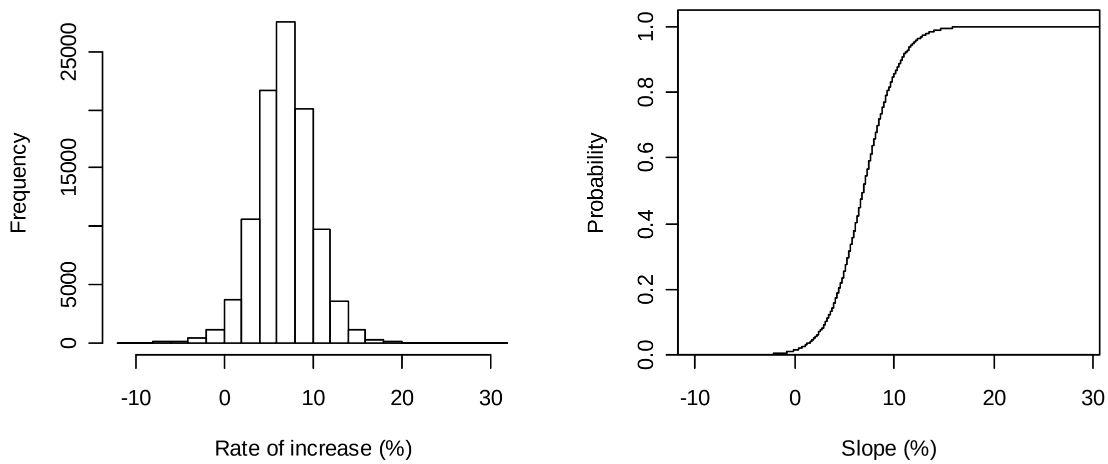

Figure 3. Posterior distribution for the rate of increase at low population size, r_0, for an "unknown" stock. The mean and standard deviation of this distribution are respectively 6.93 and 3.13.

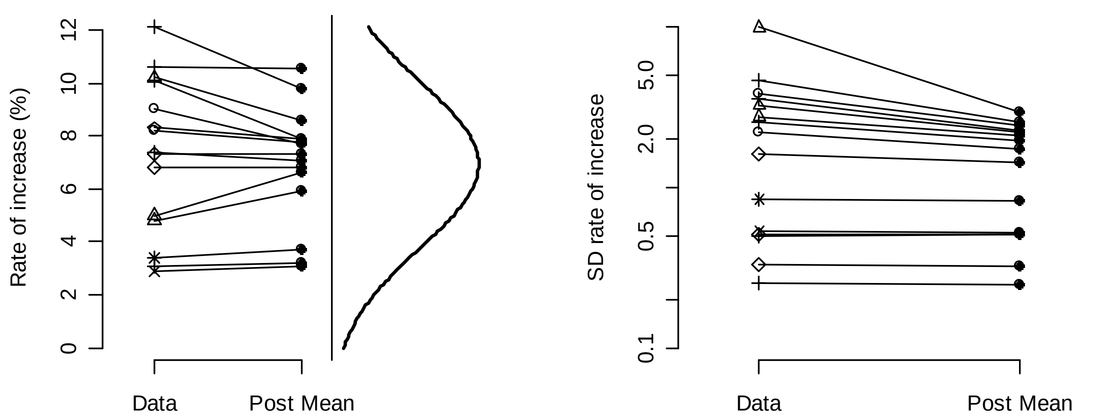

Figure 4. Left Panel - estimates of stock-specific r_0 from Table 1 and the corresponding posterior means; the distribution for the rate of increase for an unknown stock (assumed to be normal) at low population size is appended to the left panel. Right panel – estimates of the standard deviations for r_0 based on the observation error standard errors ("data") and the means of the stock-specific posteriors ("post means"). The y-axis is expressed in log-space for ease of presentation.