

## Aspects related to the estimation of variability and auto-correlation of the rate of population growth

ANDRÉ E. PUNT<sup>1</sup> AND CHERRY ALLISON<sup>2</sup>

<sup>1</sup> *School of Aquatic and Fishery Sciences, Box 35020, University of Washington, USA*

Contact e-mail: [aepunt@uw.edu](mailto:aepunt@uw.edu)

<sup>2</sup> *International Whaling Commission, The Red House, 135 Station Road, Impington, Cambridge CB4 9NP, UK.*

### ABSTRACT

Projections based on the model of Cooke (2009) are used to evaluate the extent of variation in the annual rate of population increase. This variation is found to depend on two factors: (a) the extent of variation in process error, and (b) the inherent productivity of the population (captured by the value assumed for MSYR or equivalently the parameter  $q$ ).

KEYWORDS: MSYR, VARIATION; SIMULATION

### INTRODUCTION AND METHODS

The Scientific Committee of the International Whaling Commission is conducting a review of the range of MSYR values to include in simulation trials when selecting among variants of the Revised Management Procedure (RMP). The 3<sup>rd</sup> Intersessional Workshop on the review of MSYR (IWC, 2010) calculated the extent of variation and temporal auto-correlation in the annual population rate of increase using an age-structured population dynamics model, taking account of estimates of CV and auto-correlation in calving rates (see Punt (2010) for an update to the analysis reported in IWC (2010)).

IWC (2010) recommended that similar quantities should be calculated using the age-aggregated population dynamics model developed by Cooke (2009), which has been used to evaluate the ability to estimate MSYR using survey data. This paper implements that recommendation. Table 1 lists the scenarios considered in this paper. The scenarios are a subset of those in Cooke (2009) restricted to factors which should impact the distribution for the annual rate of increase. Projections are undertaken for 2,000 years from a population initially close to zero ( $10^{-80}$ ) [so that the impact of density-dependence is negligible] and the rate of increase  $\tilde{r}_y = \ln(N_y / N_{y-1})$  computed, where  $N_y$  is the number of animals at the start of year  $y$ , computed. The summary statistics are based on the values for  $\tilde{r}_y$  for which  $N_y < 0.01K$  to ensure that the variability in annual growth rate is not impacted by density-dependence [this choice of depletion level was validated by conducting deterministic projections]

### RESULTS AND DISCUSSION

Table 2 summarizes the results of the projections in terms of the mean, standard deviation, CV and extent of temporal auto-correlation of  $\tilde{r}_y$ . Results are shown for five simulations (to assess the impact of Monte Carlo variation), and means and standard deviations over the Monte Carlo replicates are shown. A number of noteworthy features, some of which are self-evident from the structure of the age-aggregated population dynamics model, are clear from table 2: (a) the annual variation in  $\tilde{r}_y$ , as measured by the CV of  $\tilde{r}_y$ , depends critically on the value for  $q$  (and hence that for MSYR), (b) the temporal auto-correlation in  $\tilde{r}_y$  is essentially

identical to the value specified for  $\rho$ , and (c) the CV of  $\tilde{r}_y$  is not clearly linked to the value specified for  $\sigma$ , rather being substantially larger than  $\sigma$  for  $q=0.1$  and substantially smaller than  $\sigma$  for  $q=1$ . Dependence of the CV for  $\tilde{r}_y$  on the expected rate of increase at low population size was noted by Punt (2010), but the effect is much larger for the model of Cooke (2009).

## REFERENCES

- Cooke, J.G. 2009. Further analyses of the effects of environmental variability on the estimation of MSY rates. Paper SC/61/RMP13 presented to the IWC Scientific Committee, June 2008. (unpublished). 10pp.
- IWC, 2010. Report of the Third Intersessional Workshop on the Review of MSYR for Baleen Whales. IWC Document SC/62/Repxx (xxpp).
- Punt, A.E. 2010. Further analyses related to the estimation of the rate of increase for an unknown stock using a Bayesian meta-analysis. Paper SC/62/RMP3 presented to the IWC Scientific Committee, June 2010. (unpublished). 14pp.

Table 1

Summary of the scenarios considered in this paper (see Cooke (2009) for the definitions for the symbols)

| Case | $r_{\max}$ | $z$  | $q$ | $\sigma$ | $\rho$ | MSYR  | MSYL  |
|------|------------|------|-----|----------|--------|-------|-------|
| B1   | 0.1        | 2.39 | 0.1 | 0.5      | 0.5    | 0.011 | 0.509 |
| B2   | 0.1        | 2.39 | 0.4 | 0.5      | 0.5    | 0.039 | 0.538 |
| B3   | 0.1        | 2.39 | 0.9 | 0.5      | 0.5    | 0.067 | 0.589 |
| M1   | 0.1        | 2.39 | 0.1 | 0.5      | 0.9    | 0.011 | 0.509 |
| M2   | 0.1        | 2.39 | 0.4 | 0.5      | 0.9    | 0.039 | 0.538 |
| M3   | 0.1        | 2.39 | 0.9 | 0.5      | 0.9    | 0.067 | 0.589 |
| N1   | 0.1        | 2.39 | 0.1 | 1        | 0.9    | 0.011 | 0.509 |
| N2   | 0.1        | 2.39 | 0.4 | 1        | 0.9    | 0.039 | 0.538 |
| N3   | 0.1        | 2.39 | 0.9 | 1        | 0.9    | 0.067 | 0.589 |

Table 2

Summary of the results of the projections. For each projection, the mean, standard deviation, CV and lag-1 autocorrelation of the annual rate of population growth is listed.

|             | $q = 0.1$ |       |       |       | $q = 0.4$ |       |       |       | $q = 1$ |       |       |       |
|-------------|-----------|-------|-------|-------|-----------|-------|-------|-------|---------|-------|-------|-------|
|             | Mean      | SD    | CV    | Auto  | Mean      | SD    | CV    | Auto  | Mean    | SD    | CV    | Auto  |
| Cases B1-B3 |           |       |       |       |           |       |       |       |         |       |       |       |
| Sim 1       | 0.026     | 0.038 | 1.491 | 0.433 | 0.072     | 0.015 | 0.212 | 0.455 | 0.100   | 0.000 | 0.002 | 0.480 |
| Sim 2       | 0.023     | 0.044 | 1.933 | 0.465 | 0.071     | 0.016 | 0.232 | 0.471 | 0.100   | 0.000 | 0.002 | 0.364 |
| Sim 3       | 0.021     | 0.041 | 1.955 | 0.466 | 0.071     | 0.016 | 0.221 | 0.503 | 0.100   | 0.000 | 0.002 | 0.564 |
| Sim 4       | 0.024     | 0.042 | 1.731 | 0.442 | 0.072     | 0.016 | 0.226 | 0.497 | 0.100   | 0.000 | 0.002 | 0.540 |
| Sim 5       | 0.023     | 0.043 | 1.879 | 0.507 | 0.071     | 0.016 | 0.229 | 0.443 | 0.100   | 0.000 | 0.002 | 0.462 |
| Mean        | 0.023     | 0.042 | 1.798 | 0.463 | 0.071     | 0.016 | 0.224 | 0.474 | 0.100   | 0.000 | 0.002 | 0.482 |
| SD          | 0.002     | 0.002 | 0.192 | 0.028 | 0.001     | 0.000 | 0.008 | 0.026 | 0.000   | 0.000 | 0.000 | 0.078 |
| Cases M1-M3 |           |       |       |       |           |       |       |       |         |       |       |       |
| Sim 1       | 0.030     | 0.032 | 1.089 | 0.856 | 0.075     | 0.012 | 0.159 | 0.855 | 0.100   | 0.000 | 0.002 | 0.862 |
| Sim 2       | 0.025     | 0.037 | 1.497 | 0.864 | 0.072     | 0.014 | 0.194 | 0.868 | 0.100   | 0.000 | 0.002 | 0.852 |
| Sim 3       | 0.021     | 0.038 | 1.804 | 0.876 | 0.072     | 0.014 | 0.199 | 0.886 | 0.100   | 0.000 | 0.002 | 0.885 |
| Sim 4       | 0.027     | 0.049 | 1.819 | 0.920 | 0.074     | 0.017 | 0.229 | 0.911 | 0.100   | 0.000 | 0.003 | 0.916 |
| Sim 5       | 0.022     | 0.044 | 2.019 | 0.902 | 0.071     | 0.015 | 0.213 | 0.877 | 0.100   | 0.000 | 0.002 | 0.868 |
| Mean        | 0.025     | 0.040 | 1.646 | 0.884 | 0.073     | 0.014 | 0.199 | 0.879 | 0.100   | 0.000 | 0.002 | 0.876 |
| SD          | 0.004     | 0.006 | 0.363 | 0.027 | 0.001     | 0.002 | 0.026 | 0.021 | 0.000   | 0.000 | 0.000 | 0.025 |
| Cases N1-N3 |           |       |       |       |           |       |       |       |         |       |       |       |
| Sim 1       | 0.038     | 0.060 | 1.581 | 0.798 | 0.080     | 0.020 | 0.247 | 0.786 | 0.100   | 0.000 | 0.003 | 0.778 |
| Sim 2       | 0.011     | 0.105 | 9.208 | 0.829 | 0.074     | 0.029 | 0.391 | 0.808 | 0.100   | 0.000 | 0.003 | 0.799 |
| Sim 3       | 0.016     | 0.085 | 5.309 | 0.835 | 0.074     | 0.028 | 0.380 | 0.857 | 0.100   | 0.000 | 0.003 | 0.849 |
| Sim 4       | 0.024     | 0.152 | 6.396 | 0.919 | 0.042     | 0.097 | 2.301 | 0.863 | 0.100   | 0.001 | 0.006 | 0.866 |
| Sim 5       | 0.013     | 0.123 | 9.252 | 0.892 | 0.072     | 0.033 | 0.458 | 0.829 | 0.100   | 0.000 | 0.004 | 0.804 |
| Mean        | 0.021     | 0.105 | 6.349 | 0.854 | 0.069     | 0.041 | 0.755 | 0.829 | 0.100   | 0.000 | 0.004 | 0.819 |
| SD          | 0.011     | 0.035 | 3.179 | 0.050 | 0.015     | 0.031 | 0.867 | 0.033 | 0.000   | 0.000 | 0.001 | 0.037 |