SC/62/RMP3

Further Analyses Related to the Estimation of the Rate of Increase for An Unknown Stock Using a Bayesian meta-analysis

ANDRÉ E. PUNT

School of Aquatic and Fishery Sciences, Box 35020, University of Washington, USA Contact e-mail: aepunt@uw.edu

ABSTRACT
The approach developed by Punt (2010a) to construct a probability distribution for the rate of increase for an 'unknown' stock in the limit of zero population size, r_0, is tested further based on scenarios which better the reflect the data sets on which the MSYR review will be based. The results confirm that the estimates of lower percentiles of the posterior for the ratio of r_0 to the maximum demographically possible rate of increase are positively biased, with the performance of the method deteriorating with increasing levels of process error. Population projections are conducted for 13 cases to estimate the coefficient of variation and temporal correlation in the annual rate of increase. These quantities are shown to differ markedly among stocks, with the largest values for the CV of the annual rate of increase being highest for North Atlantic right and Gulf of California blue whales and lowest for SE Atlantic right whales.

KEYWORDS: MSYR, BAYESIAN META-ANALYSIS, ENVIRONMENTAL VARIATION; SIMULATION

INTRODUCTION

The Scientific Committee of the International Whaling Commission is conducting a review of the range of MSYR values to include in simulation trials when selecting among variants of the Revised Management Procedure (RMP). A table of estimates of MSYR and rates of increase at low population size was developed by IWC (2009) as a part of this review and revised by IWC (2010a). Punt (2009) outlined an approach for conducting a meta-analysis of rates of increase at low population size, r_0, for whales stocks based on the Bayesian paradigm. Punt (2010a) revised this approach to impose a lower bound of 0 for r_0 and to account for the uncertainty caused by environmental variation in population growth rates (see Appendix A).

Punt (2010a) conducted some initial tests of this method to assess whether it will work in principle. This document conducts further tests of the method based on characteristics of the (actual) data on the rates of increase in IWC (2010a).

The April 2010 MSYR Workshop (IWC, 2010b) recommended that a population dynamics model developed during the workshop (Appendix B) should be applied to data on population parameters and estimates of the extent and variability in calving rates to estimate the standard deviation and temporal auto-correlation of the rate of increase. This paper reports the results of those calculations.

METHODS

Simulation evaluation of the estimator of the distribution for the rate of growth for an 'unknown' stock.

The performance of the estimator in Appendix A is evaluated by generating simulated data sets. For computational ease, r_{max}=0.1 and z=2.39 for all data sets. Table 1 lists information for the stocks which the Scientific Committee selected as the basis for the meta-analysis of population growth at SC61. This table also lists the range of years on which the estimates are based. Table 2 uses the information in table 1, as well as the scenarios regarding the extent of variation and auto-correlation in the environmental impact on r (τ and ρ in Appendix A respectively) from Cooke (2009), to develop nine trials for testing the estimation method in Appendix A. Trial 1 represents the "best" values for the number of stocks, the period of years on which the estimate of the rate of increase is based, and the observation error standard deviation for the rate of population growth. The value for the first of these quantities is set to the actual number of stocks in table 1, while the values for the latter two parameters are set to the medians for the respective quantities in table 1. Trial 1 represents the most difficult case in terms of the values for τ and ρ so Trials 4-6 consider cases in which the extent of environmental impact on r and the temporal auto-correlation in r are lower than the values for Trial 1 (the specific choices correspond to selections by Cooke (2009)). Trials 7-9 explore the impact of changing the number of stocks, the period of years over which the rate of increase is estimated, and the extent of observation error. 500 simulations are conducted for each case.

Figure 1 shows the distributions for $r_0^{true} / r_{\max}$ for each of the three choices for α and β in table 2. The performance of the estimation method is summarized by the relative bias and relative root-mean-square-error (RMSE) for the lower 1st, 2nd, 5th, 10th, 25th and 50th percentiles for the distribution for $r_0^{true} / r_{\max}$ for an unknown stock. Table 3 lists the "true" values for these percentiles for the three choices for α and β.

Population growth rates distributions for various stocks

Table 4 lists the values for the parameters that determine the rate of increase considered in this paper. The approach of Appendix B is used to estimate the distribution for the annual rate of increase. As noted in Appendix B, the distribution for the annual rate of increase is determined by projecting a population ahead in the absence of density-dependence and recording the mean, standard deviation, CV and lag-1 autocorrelation over years 200-2,000 for $\tilde{r}_y$. These four statistics are also recorded for the "raw" calving rate $N_{y,0} / N_y^m$. Previous analyses (Punt, 2010b) have shown that the standard deviation and temporal auto-correlation in the raw calving rate will not match the pre-specified values for these quantities ($\tilde{\sigma}_f$ and $\tilde{\rho}^f$ in Table 4) if the values for σ_f and ρ^f are set to $\tilde{\sigma}_f$ and $\tilde{\rho}^f$. This occurs because of the constraints imposed by the population dynamics model (i.e. the calving rate for females which did not give birth the previous year cannot exceed 1, and females cannot give birth in consecutive years). Consequently, the values for σ_f and ρ^f are adjusted ("tuned") until the model-predicted standard deviation and temporal auto-correlation of the raw calving rate does match the pre-specified values.

RESULTS AND DISCUSSION

Simulation evaluation of the estimator of the distribution for the rate of growth for an 'unknown' stock.

Punt (2010a) showed that the method of Appendix A performs well when there are a very large number of stocks (1,000) and when the observation error standard deviation is small (~1%). However, both of these assumptions (particularly the first) are optimistic for the real data (Table 1). The biases and RMSEs for the six percentiles are shown in tables 5 and 6 respectively, and the estimates for each percentile are shown in Figures 2 and 3 for two cases (1 and 6). The estimates of the percentiles (except for the median) are positively biased. The biases are largest for case 2 (the case in which the true value for α differs the most from its prior mean) and smallest for case 3 (the case in which the true value for α is most similar to the mean of its prior). The estimates are biased by 20-30% on average for case 1, e.g., the true value for $r_0^{true} / r_{\max}$ is 0.105 and the mean of the estimates is 0.133 for the 1st percentile.

The RMSEs are lower than for case 1 when the extent of environmental impact on *r* and the temporal auto-correlation in *r* are lower than the values on which case 1 is based even though the estimator is aware of the true values for these parameters in all cases. However, performance remains better when the extent of process error is lower.

The RMSEs are lower than for case 1 for case 8 (more years on which the estimate of the rate of increase is based), but is surprisingly higher for case 7 (30 rather than 15 stocks). The latter must reflect a form of transient effect because performance is much improved when the number of stocks is very large (~1,000).

Population growth rates distributions for various stocks

The columns 'σ_f' and 'ρ^f' in table 4 list the values for the standard deviation and auto-correlation in calving rate which were needed so that the standard deviation and auto-correlation in calving rate from the model matched the pre-specified values for $\tilde{\sigma}_f$ and $\tilde{\rho}^f$. Except in one case, SE Atlantic right whales, σ_f is greater than $\tilde{\sigma}_f$. The pattern of the difference between ρ^f and $\tilde{\rho}^f$ is less clear. It should be noted that the survival rate for SE Alaska humpback whales had to be increased from its pre-specified value of 0.97 to 0.98 so that it was possible to match the pre-specified rate of increase of 0.06 without needing a calving rate in excess of 1. The possible need to adjust the value for some of the parameters in table 1 in order for the calving rate not exceed 1 was recognized by IWC (2010b).

Table 7 lists the mean, standard deviation, CV and auto-correlation coefficient for the annual rate of increase for the 13 cases in table 4. Results are shown for five applications of the model to evaluate Monte Carlo error (which seems very minor). The mean annual rate of increase is consistently lower than the input value, with the extent of difference a function of σ_f and ρ^f. The values for the CV of the annual rate of increase differ markedly among the 13 cases (lowest for SE Atlantic right and largest for North Atlantic right). The very high CV for the latter stock is attributable in part to the low value for r_0 for this stock. The levels of auto-correlation in table 7 are lower than the values assumed by Cooke (2009) and in table 2. However, the CVs in table 7 include values that are substantially larger than the largest values considered by Cooke (2009) and in table 2 (e.g. North Atlantic right and Gulf of California blue).

Comparisons between the values in table 7 and those of Cooke (2009) and in table 2 need to be made cautiously because the values for $\tilde{\sigma}_f$ and $\tilde{\rho}^f$ in table 4 do not (yet) take account of observation error in the time-series from which they were estimated, and the estimates of $\tilde{\sigma}_f$, in particular, should be positively biased (IWC, 2010b).

ACKNOWLEDGEMENTS

NOAA/NMFS/NMML are thanked for providing funding support through grant number AB133F-09-SE-4556.

REFERENCES

Cooke, J.G. 2009. Further analyses of the effects of environmental variability on the estimation of MSY rates. Paper SC/61/RMP13 presented to the IWC Scientific Committee, June 2008. (unpublished). 10pp.

IWC. 2009. Report of the MSYR Workshop. Paper SC/60/Rep5 presented to the IWC Scientific Committee, June 2008. (unpublished). 16pp.

IWC, 2010a. Report of the Sub-Committee on the Revised Management Procedure, Annex D of the Report of the Scientific Committee. *J. Cet. Res. Manage. Res.* 00:00.

IWC, 2010b. Report of the Third Intersessional Workshop on the Review of MSYR for Baleen Whales. IWC Document SC/62/Repxx (xxpp).

Punt, A.E. 2009. Constructing a posterior distribution for the rate of increase of whale stocks at low population size. IWC Document SC/F09/MSYR1 (7pp).

Punt, A.E. 2010a. A revised Bayesian meta-analysis for estimating a posterior distribution for the rate of increase for an “unknown” stock. IWC Document SC/62/A10/MSYR2 (9pp).

Punt, A.E. 2010b. Population projections under different levels of process error. Annex D to the Report of the Third Intersessional Workshop on the Review of MSYR for Baleen Whales. IWC Document SC/62/Repxx.

Appendix A
Estimating a posterior distribution for r_0 for an unknown stock

In the following $\hat{r}_i$ is the estimate of the rate of increase for stock *i*, and σ_i is the (estimate of) the observation error standard deviation for $\hat{r}_i$. Let us first define $r_{0,i}^{true}$ as the expectation of the rate of increase for stock *i* at low stock size and $r_{\max,i}$ as the maximum demographically possible rate of increase for stock *i* (assumed to be known exactly). Now, $r_{0,i}^{true} / r_{\max,i} = \chi_i$ is assumed to be beta-distributed, i.e. $\chi_i \sim Be(\alpha,\beta)$[1], and $\hat{r}_i$ is assumed to be distributed about a "realized" rate of increase subject to observation error, i.e. $\hat{r}_i \sim r_i^{real} + v_i$ where $v_i \sim N(0;\sigma_i^2)$. The realized rate of increase is related to true rate of increase, accounting for process uncertainty caused by environmental variation, i.e. the distribution of $(1+r_i^{real})^n$ is:

$$(1+r_i^{real})^{n_i} = \prod_{y=1}^{n_i} \exp\{r_{\max,i}(1-e^{-(\tau w_y - \tau^2/2)}(1-q_i)^z)\} \tag{A.1}$$

where n_i is the number of data points for stock *i*, $w_y = \rho w_{y-1} + \sqrt{1-\rho^2}\varepsilon_y$, $\varepsilon_y \sim N(0;1)$, ρ is the extent of auto-correlation in the environmental impact on *r*, and τ is the standard deviation for the environmental impact on *r*.

Now, given $q_i = (1-(1-\chi_i)^{1/z})$, *z*, ρ, and τ (assumed known) one can generate a distribution for $(1+r_i^{real})^{n_i}$ numerically. For estimation purposes, the mean of r_i^{real} can be approximated using the formula:

$$E(r_i^{real}) = \alpha_1 r_{\max,i} + \alpha_2 \chi_i + \alpha_3 (r_{\max,i})^2 + \alpha_4 (\chi_i)^2 + \alpha_5 r_{\max,i}\,\chi_i + \alpha_6 (r_{\max,i})^2 \chi_i + \alpha_7 (\chi_i)^2 r_{\max,i}$$

The standard deviation of r_i^{real}, $Var(r_i^{real})$ is approximated similarly.

The likelihood function is then:

$$L(D\,|\,\alpha,\beta) = \prod_i \int_0^1 \frac{\Gamma(\alpha,\beta)}{\Gamma(\alpha)\Gamma(\beta)} \chi_i^{\alpha-1}(1-\chi_i)^{\beta-1} \frac{1}{\sqrt{2\pi}\tilde{\sigma}_i} e^{-[\hat{r}_i - E(r_i^{real})]^2/(2\tilde{\sigma}_i^2)} d\chi_i \tag{A.2}$$

where $\tilde{\sigma}_i^2 = \sigma_i^2 + Var(r_i^{real})$

The integrals in Equation A.2 are evaluated numerically (in this case by applying the trapezoidinal rule with 100 steps). The priors for α and β are assumed to be uniform, in this paper, U[0,10].

[1] The beta distribution is selected here because it provides a flexible way to model bounded random variables.

Appendix B
Estimate the standard deviation and temporal autocorrelation in the rate of increase

The following population dynamics model forms the basis for the forecasts under different levels of variability in calving rate (and in principle survival):

$$N_{y,a} = \begin{cases} f_y\,(N_y^m - N_{y-1,0}S_{y-1}) & \text{if } a = 0 \\ N_{y-1,a-1}\,S_{y-1} & \text{if } 1 \le a < x \\ (N_{y-1,x} + N_{y-1,x-1})S_{y-1} & \text{if } a = x \end{cases} \tag{B.1}$$

where $N_{y,a}$ is the number of animals of age a at the start of year y,

N_y^m is the number of "mature" females at the start of year y:

$$N_y^m = 0.5\sum_{a=a_m}^{x} N_{y,a} \tag{B.2}$$

f_y is the calving rate (number of calves per mature female which did not calf the previous year – this number of mature females is given by $N_y^m - N_{y-1}S_{y-1})$ during year y:

$$f_y = fe^{\varepsilon_y^f - \sigma_f^2/2} \qquad \varepsilon_y^f = \rho^f\varepsilon_{y-1}^f + \sqrt{1-(\rho^f)^2}\,\eta_y^f \quad \eta_y^f \sim N(0;\sigma_f^2)\,^2 \tag{B.3}$$

f is the expected calving rate (in the absence of density-dependence),

ρ^f is the extent of auto-correlation in calving rate,

σ_f is the extent of variation in calving rate,

S_y is the survival rate during year y ($S_y = e^{-M_y}$):

$$M_y = \bar{M} + \varepsilon_y^M \qquad \varepsilon_y^M = \rho^M\varepsilon_{y-1}^M + \sqrt{1-(\rho^M)^2}\,\eta_y^M \qquad \eta_y^M \sim N(0;\sigma_M^2) \tag{B.4}$$

ρ^M is the extent of auto-correlation in natural mortality, and

σ_M is the extent of variation in natural mortality (set equal to 0 for the analyses of this paper).

The population is projected ahead for 2,000 years, and the annual rate of increase, $\tilde{r}_y = \ell\text{n}(N_y^m / N_{y-1}^m)$ is computed. The outcomes from this algorithm are the mean, standard deviation, CV and lag-1 autocorrelation over years 200-2,000 for $\tilde{r}_y$ and the "raw" calving

[2] Subject to the constraint that calving rate cannot exceed 1 (if a generated value for the calving rate exceeds 1, the value for η_y^f is generated again and this process repeated until the calving rate is less than 1).

rate $N_{y,0} / N_y^m$[3]. The value for f in Equation B.3 is not pre-specified, but is rather chosen so that the deterministic rate of increase is equal to the pre-specified value for r_0 in table 4.

[3] The raw calving rate was chosen for consistency with the approach used when analysing the data for the actual populations in table 4.

Table 1

Estimates of r_0 selected by IWC (2010a) and the associated time periods over which they were estimated.

	r_0 (%) (95% CI)	**SE**	**Time period**	**# Years**
Blue				
Central North Atlantic	9.0 (2.0, 17.0)	3.83[a]	1987-2001	15
Southern Hemisphere	8.2 (1.6, 14.8)	3.37[a]	1978/78-2003/04	26
Eastern North Pacific	3.2	1.4	1991-2005	16
Fin				
North Norway	5 (-13, 26)	9.95[a]	1998-98	11
Eastern North Pacific	4.8 (-1.6, 11.1)	3.24[a]	1987-2003	15
Humpback				
Western Australia	10.1 (0.9, 19.3)	4.69[a]	1982-94	13
Eastern Australia	10.9 (10.5, 11.4)	0.23[a]	1984-2007	24
Eastern North Pacific	6.4	0.9	1992-2003	12
Hawaii	10 (3-16)	3.32[a]	1993-2000	18
Gulf of Maine	6.3	1.2	N/A	
Gray				
Western	2.9 (1.9, 4.0)	0.54[b]	1994-2006	23
Bowhead				
Bering-Chukchi-Beaufort	3.9 (2.2, 5.5)	0.84[b]	1978-2001	24
Southern Right				
SE Atlantic	7.3 (6.6 ,7.9)	0.33[a]	1971-2003	33
SW Atlantic	6.8 (5.8 ,7.8)	0.51[a]	1971-2000	30
SE Indian	8.10 (4.48-11.83	1.88[a]	1993-2006	14

a – computed from the 95% confidence interval by dividing by 3.92

b – computed from the 90% confidence interval by dividing by 3.28

Table 2

The values for the parameters which define the simulation experiments. N is the number of years over which each stock is monitored, N_{stock} is the number of stocks for which data are available, and α and β define the true distribution for r_0^{true} / r_{max}.

Case No	$\mathbf{N_{stock}}$	**N**	τ	ρ	σ	α	β
1	15	17	1	0.9	0.014	3	3
2	15	17	1	0.9	0.014	2	4
3	15	17	1	0.9	0.014	4	2
4	15	17	1	0.5	0.014	3	3
5	15	17	0.5	0.9	0.014	3	3
6	15	17	0.5	0.5	0.014	3	3
7	30	17	1	0.9	0.014	3	3
8	15	34	1	0.9	0.014	3	3
9	15	17	1	0.9	0.028	3	3

Table 3

The true percentiles for r_0^{true} / r_{max} for the three choices for α and β

α, β	**Percentiles**					
	0.01	**0.02**	**0.05**	**0.1**	**0.25**	**0.5**
3, 3	0.105	0.134	0.189	0.246	0.360	0.500
2, 4	0.033	0.047	0.077	0.113	0.194	0.314
4, 2	0.220	0.266	0.343	0.416	0.546	0.686

Table. 4

Values for the parameters of the population dynamics model (tables 2 and x of SC/62/Repx). $\tilde{\sigma}_f$ and $\tilde{\rho}^f$ are respectively the values for the standard deviation and temporal auto-correlation in the data type concerned based on the analyses in Annex C of SC/62/Repx and σ_f and ρ^f are the "tuned" values for these parameters (selected so that the output of the population dynamics model matches the values for $\tilde{\sigma}_f$ and $\tilde{\rho}^f$).

Stock	Calving Data Type	S	a_m	r_0	$\tilde{\sigma}_f$	$\tilde{\rho}^f$	σ_f	ρ^f
BCB bowhead	Proportion	0.99	22	0.04	0.581	0.075	0.736	0.563
Eastern gray	Proportion	0.98	7	0.06	0.484	0.362	0.614	0.726
Gulf of Maine humpback	Interval	0.955	7	0.065	0.161	0.197	0.260	0.838
Gulf of Maine humpback	Proportion	0.955	7	0.065	0.454	-0.749	0.555	-0.641
Gulf St. Lawrence humpback	Interval	0.982	12&	0.065	0.236	0.283	0.392	0.853
Gulf St. Lawrence humpback	Proportion	0.982	12&	0.065	0.859	-0.494	1.318	-0.524
North Atlantic right	Proportion	0.96	9	0.01	0.416	0.160	0.419	0.373
North Atlantic right	Interval	0.96	9	0.01	0.150	0.609	0.166	0.716
SE Alaska humpback	Interval	0.98*	12	0.06	0.179	0.410	0.266	0.853
SE Alaska humpback	Proportion	0.98*	12	0.06	0.224	0.121	0.310	0.741
SE Atlantic right	Proportion	0.99	8&	0.073	0.085	-0.336	0.080	0.084
SW Atlantic right	Proportion	0.98	9.1&	0.068	0.321	-0.151	0.379	0.481
Gulf of California blue	Proportion	0.975	10&	0.07	0.915	-0.544	1.544	-0.735

& Values given in table 2 in SC/62/RepX for a_m were rounded to nearest whole age and those given as x+ were to set to age x

* Increased from 0.97 (see text for detail).

Table 5

Biases (expressed relative to the true percentile)

Case No	**Percentiles**					
	0.01	**0.02**	**0.05**	**0.1**	**0.25**	**0.5**
1	0.268	0.311	0.305	0.270	0.187	0.111
2	1.032	1.006	0.902	0.771	0.555	0.360
3	0.065	0.094	0.097	0.084	0.048	0.020
4	0.139	0.173	0.163	0.132	0.060	-0.006
5	0.164	0.191	0.173	0.139	0.062	-0.006
6	0.194	0.209	0.184	0.147	0.067	-0.003
7	0.369	0.391	0.363	0.319	0.226	0.142
8	0.133	0.185	0.196	0.175	0.112	0.050
9	0.089	0.156	0.184	0.174	0.121	0.067

Table 6
Root mean square errors (expressed relative to the true percentile)

Case No	**Percentiles**					
	0.01	**0.02**	**0.05**	**0.1**	**0.25**	**0.5**
1	0.891	0.812	0.664	0.542	0.370	0.243
2	2.402	2.087	1.642	1.302	0.867	0.547
3	0.488	0.432	0.347	0.285	0.201	0.140
4	0.652	0.579	0.456	0.363	0.239	0.161
5	0.632	0.561	0.439	0.346	0.224	0.150
6	0.596	0.520	0.399	0.309	0.193	0.129
7	0.912	0.816	0.654	0.525	0.350	0.222
8	0.745	0.677	0.553	0.449	0.302	0.196
9	0.803	0.737	0.612	0.507	0.357	0.244

Table 7
Distribution statistics for the rate of increase in the absence for density-dependence for the scenarios in table 4.

r_0	Mean	SE	CV	Auto	r_0	Mean	SE	CV	Auto
		BCB bowhead					Eastern gray		
0.040	0.035	0.026	0.749	0.055	0.060	0.052	0.032	0.617	0.253
0.040	0.036	0.026	0.734	0.039	0.060	0.054	0.032	0.597	0.255
0.040	0.036	0.026	0.715	0.049	0.060	0.054	0.031	0.583	0.274
0.040	0.036	0.026	0.728	0.039	0.060	0.054	0.033	0.605	0.253
0.040	0.036	0.026	0.732	0.074	0.060	0.053	0.033	0.622	0.259
		Gulf of Maine humpback					Gulf of Maine humpback		
0.065	0.060	0.015	0.254	-0.024	0.065	0.058	0.049	0.843	-0.787
0.065	0.061	0.015	0.248	-0.009	0.065	0.058	0.046	0.806	-0.740
0.065	0.061	0.015	0.239	-0.052	0.065	0.058	0.048	0.825	-0.758
0.065	0.061	0.015	0.246	-0.035	0.065	0.058	0.045	0.786	-0.746
0.065	0.060	0.016	0.256	0.017	0.065	0.058	0.046	0.799	-0.758
		Gulf of St Lawrence humpback					Gulf of St Lawrence humpback		
0.065	0.059	0.017	0.285	0.109	0.065	0.045	0.055	1.214	-0.519
0.065	0.060	0.016	0.272	0.150	0.065	0.045	0.054	1.193	-0.487
0.065	0.060	0.016	0.265	0.088	0.065	0.046	0.055	1.194	-0.511
0.065	0.060	0.016	0.272	0.122	0.065	0.045	0.053	1.169	-0.478
0.065	0.060	0.017	0.286	0.160	0.065	0.045	0.053	1.159	-0.508
		North Atlantic right					North Atlantic right		
0.010	0.009	0.019	2.282	0.074	0.010	0.009	0.007	0.709	0.486
0.010	0.010	0.021	2.124	0.120	0.010	0.010	0.007	0.697	0.521
0.010	0.010	0.020	2.068	0.134	0.010	0.010	0.007	0.692	0.539
0.010	0.009	0.020	2.138	0.124	0.010	0.010	0.007	0.709	0.534
0.010	0.010	0.020	2.115	0.113	0.010	0.010	0.007	0.707	0.540
		SE Alaska humpback					SE Alaska humpback		
0.060	0.058	0.012	0.208	0.202	0.060	0.057	0.016	0.286	-0.003
0.060	0.059	0.012	0.201	0.207	0.060	0.058	0.017	0.285	-0.023
0.060	0.059	0.012	0.204	0.226	0.060	0.058	0.016	0.275	0.040
0.060	0.059	0.012	0.208	0.212	0.060	0.058	0.016	0.281	0.013
0.060	0.058	0.012	0.214	0.221	0.060	0.058	0.017	0.289	0.023
		SE Atlantic right					SW Atlantic right		
0.073	0.073	0.007	0.096	-0.405	0.068	0.065	0.027	0.417	-0.217
0.073	0.073	0.007	0.097	-0.354	0.068	0.065	0.027	0.407	-0.201
0.073	0.073	0.007	0.094	-0.347	0.068	0.065	0.026	0.401	-0.200
0.073	0.073	0.007	0.095	-0.352	0.068	0.065	0.027	0.411	-0.218
0.073	0.073	0.007	0.097	-0.411	0.068	0.065	0.027	0.411	-0.212
		Gulf of California blue							
0.070	0.042	0.061	1.452	-0.554					
0.070	0.042	0.061	1.443	-0.549					
0.070	0.043	0.063	1.449	-0.566					
0.070	0.042	0.060	1.423	-0.546					
0.070	0.042	0.060	1.423	-0.549					

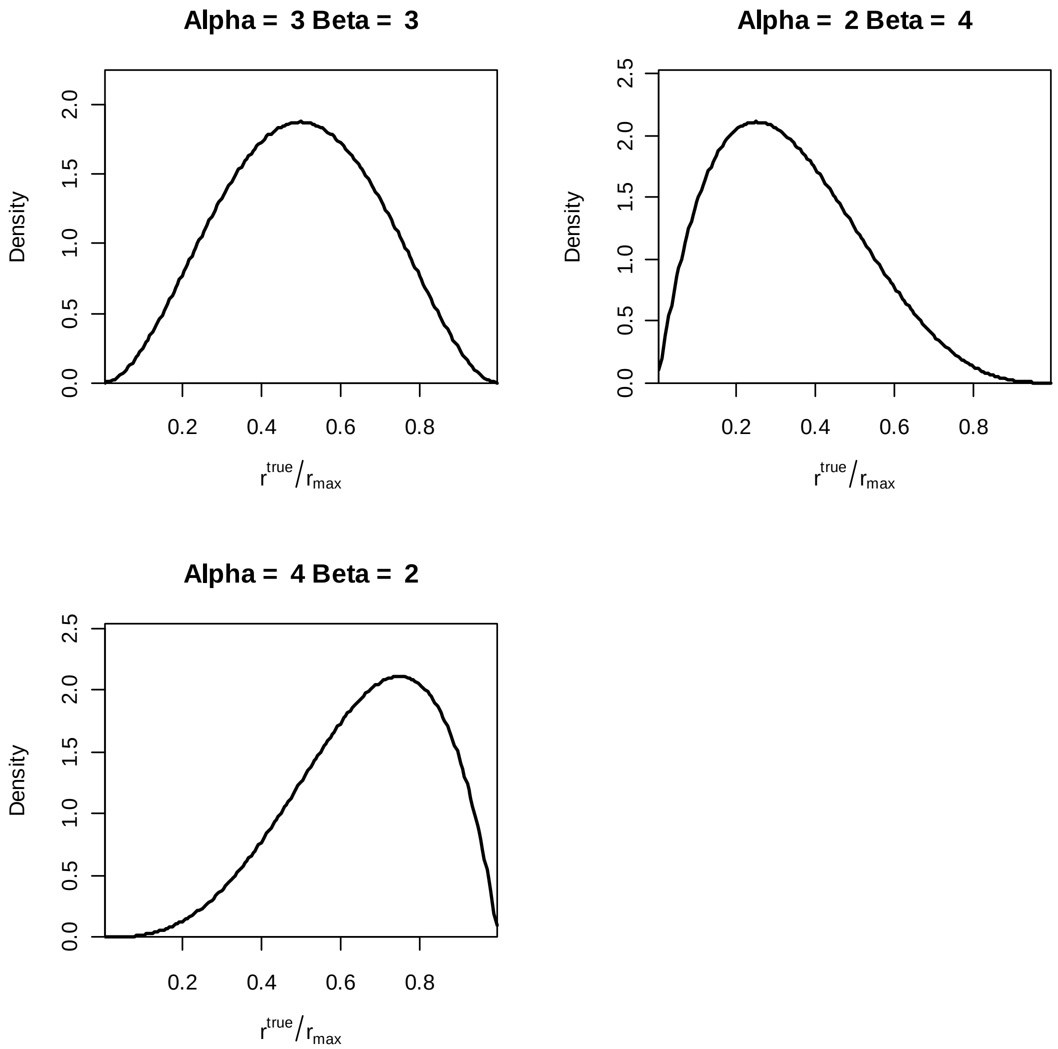

Figure 1. Distributions for $r_0^{true} / r_{\max}$ for the three choices for α and β considered in the simulation study.

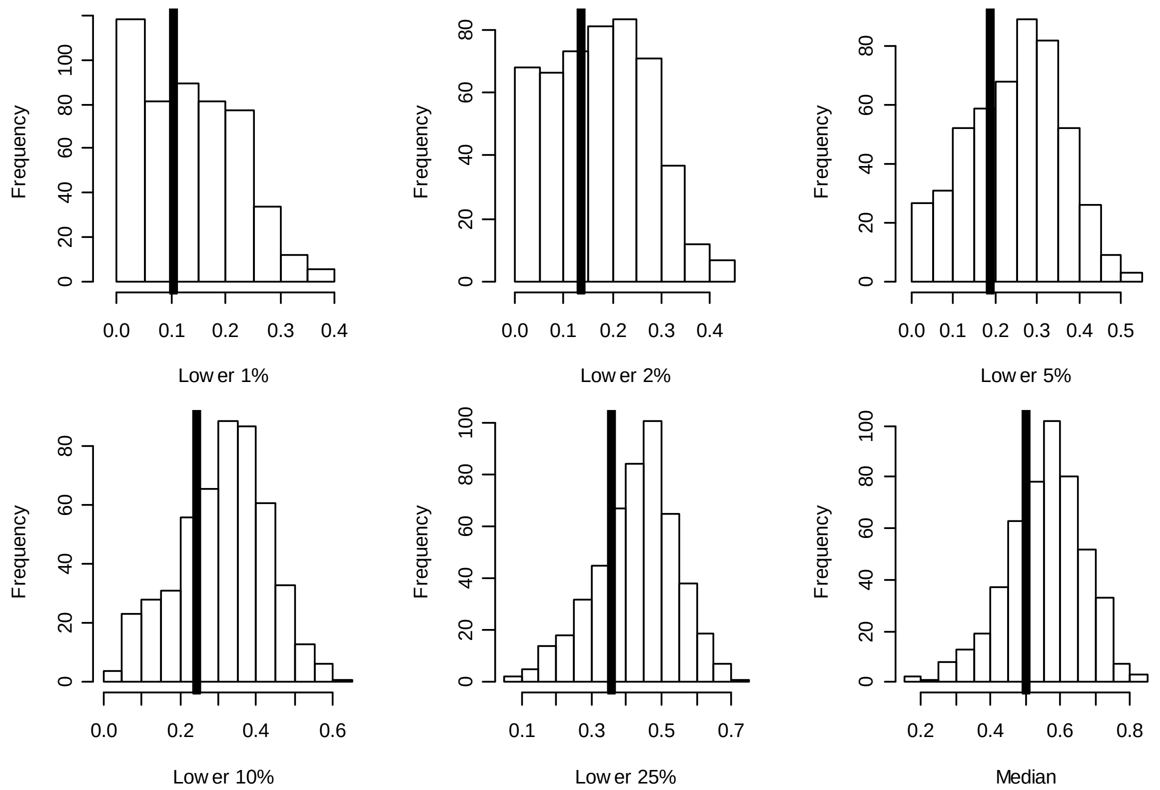

Figure 2. Estimates of the lower percentiles of the distribution for $r_0^{true} / r_{\max}$ for case 1. The solid line denotes the true percentile.

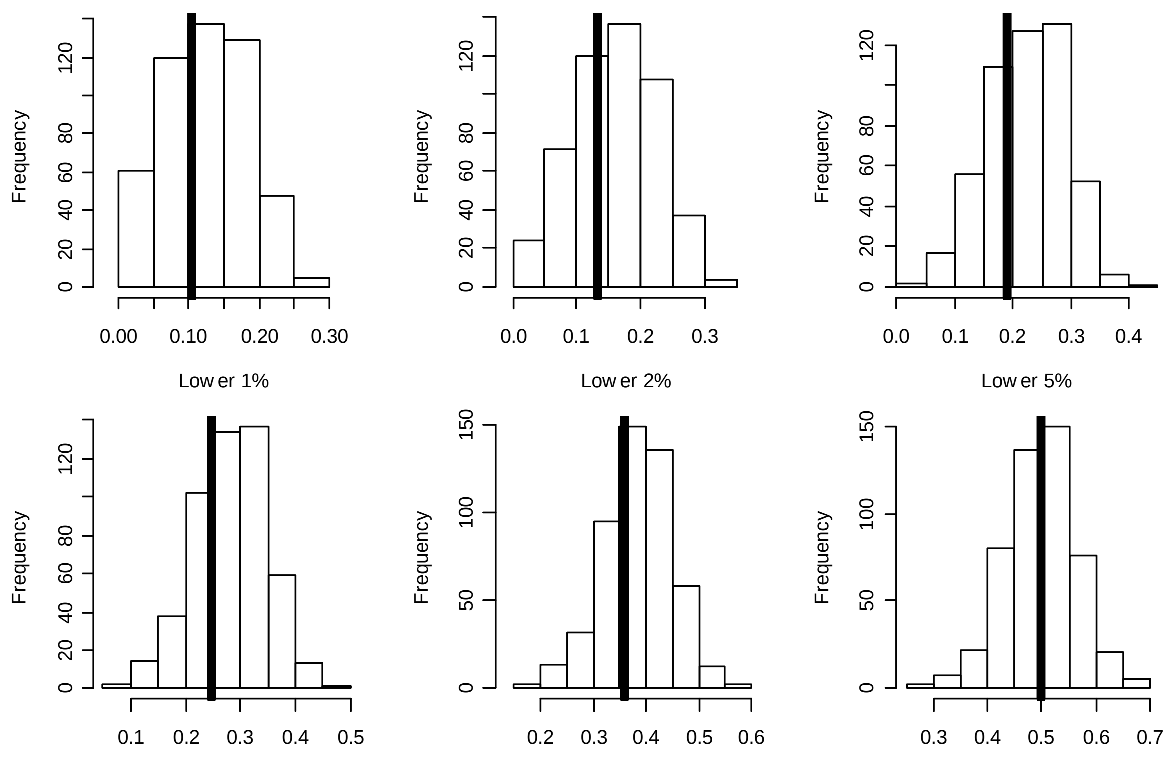

Figure 3. Estimates of the lower percentiles of the distribution for $r_0^{true} / r_{\max}$ for case 6. The solid line denotes the true percentile.