On population dynamics of eastern Canada - West Greenland bowhead whales

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ABSTRACT

We use recent abundance estimates, historical catches starting from 1719, and an age- and sex-structured population model to perform Bayesian assessments of bowhead whales (*Balaena mysticetus*) in eastern Canada - West Greenland. It is examined if the population dynamics is best described by density regulated growth, with perturbed populations returning monotonically towards an equilibrium state, or by inertia dynamics, where populations typically return through damped cycles. For bowhead whale in Eastern Canada - West Greenland there is substantial statistical support for the acceptance of inertia dynamics and the rejection of density regulated growth. The abundance declined from a population dynamic equilibrium in 1719 with 30000 (90% CI:24000-35000) individuals to a maximal depletion with 1700 (90% CI:510-4900) individuals in 1888. It can be expected that the population increase to a projected abundance estimate with 10000 (90% CI:5200-20000) individuals in 2020 (assuming yearly post 2010 catches of 5). We estimate a 2011 depletion ratio of 0.29 (90% CI:0.15-0.58) and a yearly replacement of 150 (90% CI:52-450) individuals. For the alternative stock hypothesis of a Baffin Bay - Davis Strait population, we estimated that the abundance declined from a population dynamic equilibrium with 34000 (90% CI:23000-40000) individuals in 1719 to a maximal depletion of 3400 (90% CI:590-8500) individuals in 1888. It can be expected that the population increase to a projected abundance of 9100 (90% CI:4500-18000) in 2020 (assuming yearly post 2010 catches of 3). We estimate a 2011 depletion ratio of 0.25 (90% CI:0.13-0.52) and a yearly replacement of 70 (90% CI:14-330) individuals.

INTRODUCTION

In this paper we build population models in order to examine the population dynamics of bowhead whales (*Balaena mysticetus*) in eastern Canada - West Greenland, including the alternative hypothesis of an assumed Baffin Bay - Davis Strait population. The modelling framework is sex and age-structured, and it is used to reconstruct historical trajectories for a shorter (1970 to 2020) and longer (1719 to 2020) time period from abundance and catch data.

The underlying dynamics is assumed to be either unchecked exponential growth, density regulated growth, or inertia dynamics (Ginzburg 1998; Witting 2000, 2002; Ginzburg

Year	I_a	N_b
1981	11 (100)	–
1982	4 (100)	–
1990	17 (100)	–
1991	28 (58)	–
1998	42 (45)	–
2002	–	6340 (38)

Table 1: **Abundance** estimates with CV in parenthesis (given in %). I_a is sighting rates from aerial surveys in West Greenland (mature animals). N_b is an agree estimate from 2002 for Baffin Bay and Davis Strait (1+ component). Data from Heide-Jørgensen et al. (2007), Heide-Jørgensen et al. (2008), and Givens et al. (2009).

and Colyvan 2004). Inertia dynamics can show a continuum of behaviour from the monotonic increase of traditional density regulated growth, over damped, to stable cyclic behaviour. Exponential growth is useful for estimating the trend over shorter time periods, while density regulated growth and inertia dynamics are useful for examining the long-term behaviour of populations.

Our modelling is based on Bayesian statistics (Berger 1985; Press 1989), which is particularly useful when faced with limited or uncertain information. Major data uncertainties are often associated with life-history estimates, imprecise abundance estimates, additional variation in time-series of abundance estimates, and catch histories with uncertain loss and reporting rates. Our description of the dynamics aim to incorporate these uncertainties, and we use Bayes factor ratios in an attempt to identify the population dynamic model/s that provide the best description of the dynamics.

METHOD

Abundance data

The available abundance data are listed in Table 1.

The population dynamic models are fitted to the winter abundance from the Disko Bay area in West Greenland and the summer abundance from the eastern Canadian Arctic. These are sighting rates from aerial surveys in West Greenland (mature animals) [denoted I_a, from Heide-Jørgensen et al. (2007)] and an agree estimate from 2002 for Baffin Bay and Davis Strait [N_b, from Heide-Jørgensen et al. (2008) and Givens et al. (2009)]. The data provide the best available estimates of the trend and absolute abundance of an eastern Canada West Greenland population of bowhead whales, as well of an assumed Baffin Bay Davis Strait population. Trend information is given by the aerial sighting rates from the core area of bowheads in West Greenland during late winter early spring, as obtained from Heide-Jørgensen et al. (2007). They include the 1981, 1982, 1990 and 1998 estimates, but not the sighting rate from 2006, which was obtained from a different survey platform. The sighting rates are applied here to the mature component of the population, because data on body length suggests that it is primarily large mature whales without calves that occupy in the Disko Bay area (Heide-Jørgensen et al. 2010).

Catch data

Catch histories are shown in Figure 1, and listed in full length in the supplement part of the paper.

Eastern Canada - West Greenland

The population dynamic modelling use 2 catch histories. These are a low catch history and a high catch history for eastern Canada - West Greenland . The catch histories are the estimates provided by Higdon (2010). The low catch history exclude the low quality data, and it begins in 1719 when data become availabe for the Dutch harvest in the Davis Strait. The high catch history include estimates from all data sources, and it begins in 1530 with the Basque whaling off Labrador, but is applied here only from 1719.

Baffin Bay - Davis Strait

The population dynamic analysis is based on 2 catch histories. These are a low catch history and a high catch history for Baffin Bay - Davis Strait .

The low catch history excludes the low quality data, and it begins in 1719 when data become availabe for the Dutch harvest in the Davis Strait. It is the catch history sum from Higdon (2010), but with catches from the Hudson Bay subtracted. The Hudson Bay commercial harvest occurred from 1860 to the early 1900s, and only two nations were involved (Scottish and Americans). The Hudson's Bay Company attempted some limited whaling between 1767 and 1772, but they had little success and only six whales were taken.

The estimated Inuit harvest was broken down by various areas and regions. It is assumed that after 1860 most whales harvested by Inuit would have products traded (mainly baleen) to the Hudsons Bay Company or the whalers, and would therefore be included in the commercial catches. There were also minor and sporadic harvests after commercial whaling ended. Recent Nunavik harvests were landed in Hudson Strait in August, and assumed to be whales from either Hudson Bay or Foxe Basin.

The high catch history include estimates from all data sources, and it begins in 1530 with the Basque whaling off Labrador, but is applied here only from 1719.

Population dynamics

Three different models of population dynamics were applied. A model of exponential growth was used as the simplest realistic population model to estimate trends and production potentials, assuming that a stable yearly production is realistic on the short time-scale from 1970 to 2020. A second model of density regulated growth was applied to allow for estimates of current and historical depletion levels, should the population dynamics under constant environmental conditions be monotonically returning towards equilibrium.

A third model of inertia dynamics was applied to allow for estimates of depletion levels should the dynamics be damped to stable cyclic.

Let x be the maximum lumped age-class. Let the number $N_{a,t+1}^{m/f}$ of males (m) and females (f) in age-classes $0 < a < x$ in year $t+1$ be

$$N_{a+1,t+1}^{m/f} = p_a^{m/f} N_{a,t}^{m/f} - c_{a,t}^{m/f} \tag{1}$$

and the number of animals in age-class x be

$$N_{x,t+1}^{m/f} = p_x^{m/f} N_{x,t}^{m/f} + p_{x-1}^{m/f} N_{x-1,t}^{m/f} - c_{x,t}^{m/f} - c_{x-1,t}^{m/f} \tag{2}$$

where $p_a^{m/f}$ is the age specific survival rate of males/females, and $c_{a,t}^{m/f}$ is the age specific catch of males/females in year t. The age and gender (g) dependent survival rates $p_a^g = p\tilde{p}_a^g$ are given as a product between a survival scalar p and a relative ($0 < \tilde{p}_a^g \leq 1$) survival rate, with the sex and age structure of the relative survival rates being given in Table 2. The age and gender specific catches $c_{a,t}^{m/f} = c_t^{m/f} \tilde{c}_a^{m/f}$ in year t is given as a product between the total catch of males/females ($c_t^{m/f}$), as specified by the catch history, and the age-specific catch selectivity ($\tilde{c}_a^{m/f}$), as specified separately for males and females in Table 3.

The number of females and males in age-class zero is $N_{0,t}^f = \vartheta N_{0,t}$ and $N_{0,t}^m = (1 - \vartheta)N_{0,t}$, where ϑ is the fraction of females at birth, and

$$N_{0,t} = \sum_{a=a_m}^{x} B_{a,t} \tag{3}$$

where a_m is the age of the first reproductive event and $B_{a,t}$, the number of births from females in age class a, is

$$B_{a,t} = b_{a,t}\, \tilde{b}_a\, M_{a,t}^f \tag{4}$$

where $b_{a,t}$ is the birth rate in year t for age-class a females should they be at their age-specific reproductive peak, $0 < \tilde{b}_a \leq 1$ is the relative age-specific birth rate (given in Table 4), and $M_{a,t}^f$ is the number of mature females in age-class a in year t, defined as

$$M_{a,t}^f = \begin{cases} 0 & \text{if } a < a_m \\ N_{a,t}^f & \text{if } a \geq a_m \end{cases} \tag{5}$$

Let $b_{a,t}$ be

$$\begin{aligned} b_{a,t} &= b && \text{for exponential growth} \\ b_{a,t} &= b^* + [b_{\max} - b^*][1 - (\hat{N}_t/\hat{N}^*)^\gamma] && \text{for density regulated growth} \\ b_{a,t} &= \min[b_{\max}, \dot{b}_a(\hat{N}_t/\hat{N}^*)^{-\gamma}] && \text{for inertia dynamics} \end{aligned} \tag{6}$$

where b is a constant birth rate, b^* is the birth rate at population dynamic equilibrium (assuming zero catch and equilibrium denoted by $*$), $b_{\max}$ is the maximal birth rate, $\dot{b}_a$ is the average intrinsic birth rate for females in age-class a, γ is the density dependence parameter, and the abundance component that imposes density dependence is the one-plus component

$$\hat{N}_t = \sum_{a=1}^{x} N_t^f + N_t^m \tag{7}$$

The average intrinsic fecundity rate of newborns, $\dot{b}_{0,t}$, is expected to be a function of the intrinsic fecundity rates of the parents and the natural selection that is imposed by the density dependent ecology. This selection pressure is, in fact, the major reason why we may expect non-negligible sized organisms with sexual reproduction between males and females to exist at all (Witting 2008). Following Witting (2000), we can approximate the selection response as

$$\dot{b}_{0,t+1} = \frac{\sum_{a=a_m}^{x} \tilde{b}_a N_{a,t} \ \min[b_{\max}, \dot{b}_{a,t}(\hat{N}_t/\hat{N}^*)^{-\iota}]}{\sum_{a=a_m}^{x} \tilde{b}_a N_{a,t}} \tag{8}$$

where ι defines the between generation response to the natural selection pressure. This response may include, but is not limited to, genetic driven changes. It may also include responses by epigenetic inheritance, maternal effects, and selection induced between generation changes in the way that the individuals interact with one-another and the environment. Assuming that there is no change in the intrinsic fecundity rate of a cohort over time, $\dot{b}_{a+1,t+1} = \dot{b}_{a,t}$ and

$$\dot{b}_{x,t+1} = \frac{\dot{b}_{x,t}(p_x^f N_{x,t}^f - c_{x,t}^f) + \dot{b}_{x-1,t}(p_{x-1}^f N_{x-1,t}^f - c_{x-1,t}^f)}{N_{x,t+1}^f} \tag{9}$$

Given a stable age-structure and no catch, let, for a traditional model of exponential or density regulated growth, λ be a constant defined by $\hat{N}_{t+1} = \lambda \hat{N}_t$. The sustainable yield is then sy $= \hat{N}(\lambda - 1)$, and for the density regulated model there is an optimum $\partial \text{sy}/\partial \hat{N} = 0$; the maximum sustainable yield (msy) at $\hat{N}_{\text{msy}}$, also known by the maximum sustainable yield rate (msyr $=$ msy$/\hat{N}_{\text{msy}}$) and the maximum sustainable yield level (msyl $= \hat{N}_{\text{msy}}/\hat{N}^*$). For inertia dynamics, however, the intrinsic growth rate is an initial condition, unlike the fixed parameter for exponential and density regulated growth. For a given abundance, this implies that there is no constant λ to define a constant sustainable yield. Hence, there is no single abundance curve of sustainable yields and, thus, no easily defined maximum of sustainable yield. For any single abundance at a given time the yield that will leave the abundance unchanged for the next generation may, dependent upon initial conditions and time, be any value within a range of both positive and negative numbers.

Assessment models

The population dynamic description is based on the 4 assessment models that are described in this subsection.

Eastern Canada - West Greenland

Short-term exponential (e): Short-term (1970-2020) model based on exponential growth. Only very few recent catches will distinguish this model from a short-term model for an assumed Baffin Bay - Davis Strait population.

Density regulated growth (d): Long-term (1719-2020) model based on density regulated growth.

Inertia dynamics (i): Long-term (1719-2020) model based on inertia dynamics.

Baffin Bay - Davis Strait

Only a single assessment model is applied: **Inertia dynamics (iB)**: Long-term (1719-2020) model based on inertia dynamics.

Statistical methods

The assessment models were fitted to data by projecting the population under the influence of the historical catches, with the initial abundance reflecting, dependent upon the model, a pre-harvested population in dynamic equilibrium or an abundance prior for the first year of the iteration. A Bayesian statistical method (e.g, Berger 1985; Press 1989) was used, and posterior estimates of model parameters and other management related outputs were calculated. This implied an integration of the product between a prior distribution for each parameter and a likelihood function that links the probability of the data to the different parameterisations of the model.

Prior distributions

The values and prior distributions of the different parameters for all the assessment models are listed in Table 5.

The population dynamic growth rate was given a uniform prior from -0.07 to 0.07 in the exponential model. For the models of density regulated growth and inertia dynamics, the prior on the maximal population dynamic growth rate was set to resemble the estimate of current growth in the Bering-Chukchi-Beaufort Seas population of bowhead whales. Zeh and Punt (2005) estimated this growth rate to 3.4% (95% CI: 1.7%–5%) per year, and it is applied here as a beta distribution ($a = 14.8$, $b = 421$).

In accordance with the IWC assessments of the Bering-Chukchi-Beaufort Seas population of bowhead whales (IWC 2003), the prior on the age of the first reproductive event (a_m) was uniform from 14 to 26 years of age, and the maximal birth rate (or realised for the exponential model) was uniform from 0.25 to 0.50, reflecting an birth interval from two to four years. The female fraction at birth was set to 0.5, and the 1+-survival rate was estimated from the other life history parameters, given a uniform first year survival rate prior as a fraction 0.6 to 1.0 of the 1+-survival rate.

The density regulation parameter (γ) was given a uniform prior from 1.5 to 5 in the density regulated models, to mimic a msyl in the range from 0.5 to 0.7. A log uniform prior was set by trial and error for the density regulation (γ) and the inertia (ι) parameters of the inertia model to provide the best long-term fits of the model to the abundance data, given that the population was allowed to overshoot the pre-harvest abundance only once since 1664. For the inertia model the initial condition on the growth rate was set to be zero growth of an assumed population dynamic equilibrium prior to the first catches. The assumption of a pre-harvested population in dynamic equilibrium was also applied to the density regulated model. For the model of exponential growth the projection was initialised with a stable-age structure.

A log uniform prior was set by trial and error for the abundance bias of the sighting rate estimates from West Greenland.

A uniform prior from zero to one was set on a catch history selection parameter c_h, with the applied catch history $c = c_h(c_h - c_l) + c_l$ representing a linear scaling between the low and the high catch history (Figure 1).

Bayesian integration

The Bayesian integration was obtained by the sampling-importance-resampling routine (Jeffreys 1961; Berger 1985; Rubin 1988), where n_s random parameterisations θ_i $(1 \leq i \leq n_1)$ are sampled from an importance function $h(\theta)$. This function is a probability distribution function from which a large number, n_s, of independent and identically distributed draws of θ can be taken. $h(\theta)$ shall generally be as close as possible to the posterior, however, the tails of $h(\theta)$ must be no thinner (less dense) than the tails of the posterior (Oh and Berger 1992). For each drawn parameter set θ_i the population was projected from the first year with a harvest estimate to the present. For each draw an importance weight, or ratio, was then calculated

$$w(\theta_i) = \frac{L(\theta_i)p(\theta_i)}{h(\theta_i)} \tag{10}$$

where $L(\theta_i)$ is the likelihood given the data, and $h(\theta_i)$ and $p(\theta_i)$ are the importance and prior functions evaluated at θ_i. In the present study the importance function is set to the joint prior, so that the importance weight is given simply by the likelihood. The n_s parameter sets were then re-sampled n_r times with replacement, with the sampling probability of the ith parameter set being

$$q_i = \frac{w(\theta_i)}{\sum_{j=1}^{n_s} w(\theta_j)} \tag{11}$$

This generates a random sample of the posterior distribution of size n_r.

The method of de la Mare (1986) was used to calculate the likelihood L under the assumption that observation errors are log-normally distributed (Buckland 1992)

$$L = \prod_i \prod_t \exp\left(-\frac{[\ln(\hat{N}_{i,t}/\beta_i N_t)]^2}{2\text{CV}_{i,t}^2}\right) / \text{CV}_{i,t} \tag{12}$$

where $\hat{N}_{i,t}$ is the point estimate of the ith set of abundance data in year t, $\text{CV}_{i,t}$ is the coefficient of variation of the estimate, N_t is the simulated abundance, and β_i a bias term with is set to one for absolute abundance estimates.

If the importance function is adequately specified, the mean of the importance sample for each parameter should approach the mean from the true posterior distribution, given a sufficiently large sample. To illustrate whether the sampled posterior quantities can be assumed to be representative of the true posterior distribution, convergence diagnostics were calculated. One such diagnostic is the maximum importance weight of a parameter set relative to the total summed importance weight over all n_s draws. McAllister et al. (2001) suggest that the maximum importance weight needs to have dropped below 1% of the total sum. And in line with Wade (2002), we also calculated the total number of unique parameter sets in the resample of n_r parameter sets, as well the maximum number of occurrences of a unique parameter set in the resample.

Models that are based on the same data are compared by Bayes factor K (Jeffreys 1961; Kass and Raftery 1995), in order to investigate if some models provide better descriptions of the data than others. The factor is calculated here as the ratio of the harmonic means of the likelihoods in the posterior distributions of the two models.

RESULTS

Sample and resample statistics are given in Table 6. The maximum importance weight of a parameter set relative to the sum of importance weights for all the sampled sets was between 203% (density regulated growth model) and 498% (short-term exponential model) across all models. The proportion of unique parameter sets in the resample of a model was between 44% (inertia dynamics model) and 69% (short-term exponential model), and the maximum number of occurrences of a unique paramater set in the resample between 8 (short-term exponential model) and 24 (density regulated growth model).

Posterior distributions

The realised prior and posterior distributions are shown in Figures 2 to 5. With n being the number of bin intervals for the distributions, and $p_{r,i}$ and $p_{s,i}$ being density weight of the prior and the posterior at the ith bin, $u = \frac{1}{n} \sum_{i=1}^{n} \frac{|p_{s,i} - p_{r,i}|}{p_{s,i} + p_{r,i}}$ gives the updating of the posterior by the data, with $u = 0$ representing no updating and complete overlap between the two distributions, and $u = 1$ representing no overlap and a complete updating. Apart from being well updated a successful posterior should also be well bounded, with the posterior/prior weight-ratio ($w_i = p_{s,i}/p_{r,i}$) at the lower ($i = 0$) and upper ($i = n$) limits of the distributions approaching zero.

Because of the type of biological information available in abundance data, it is only for the abundance parameters (N_0, N^*) that we will set up some minimum criterion for an acceptable model. Only models with a well updated posterior ($w_i < 0.5$ and $u > 15\%$) for the abundance will be taken as an acceptable description of a population. Owing to the presence of absolute abundance estimates, we should expect well updated abundance parameters. None of the models failed to pass the minimum criterior for acceptence (details given for each model below).

Relating to the other parameters, we cannot expect the posterior distributions of the life-history parameters (p, p_0, b, a_m) to be well updated by the available data, but we might expect some updating of the growth rate parameters (r, msyr) owing to the time-series of abundance data.

Eastern Canada - West Greenland

For the **short-term exponential** (e) model in Figure 2 there is a strong (60%) updating of the initial abundance (N_0); it is very clear that the prior is wider than the posterior in both ends. The exponential growth rate (r) is substantially (48%) updated; the parameter is badly defined to the right where the posterior is clearly wider than the prior. There is a weak (24%) updating of the life-history parameters (p, p_0, b, a_m). The yearly survival (p) is strongly (52%) updated, but the prior is narrower than the posterior to the right. The first year survival (p_0) has hardly any (10%) updating; the prior is slightly narrower than the posterior to the right. There is hardly any (12%) updating of the birth rate (b); the posterior is slightly wider than the prior to the right. The age of the first reproductive event (a_m) has a weak (20%) updating, and the posterior is slightly wider than the prior to the left. The updating of the abundance estimate bias (β_a) is strong (57%); it is very clear that the prior is wider than the posterior in both ends.

For the **density regulated growth** (d) model in Figure 3 there is a strong (66%) updating of the population dynamic equilibrium abundance (N^*), and the prior is very clearly wider than the posterior in both ends. The exponential growth rate (r) has a weak (21%) updating, and the parameter is not well defined because the prior is slightly narrower than the posterior in both ends. The maximum sustainable yield rate (msyr) is weakly (24%) updated, and the parameter is very badly defined because the posterior is wider than the prior in both ends. There is hardly any (9%) updating of the life-history parameters (p, p_0, b, a_m). The yearly survival (p) is weakly (19%) updated; the parameter is badly defined because it is clear that the prior is narrower than the posterior, especially to the left. The updating of the first year survival (p_0) is very weak (7%), and the posterior is slightly wider than the prior to the right. The birth rate (b) has hardly any (6%) updating, and the posterior is slightly wider than the prior to the right. The updating of the age of the first reproductive event (a_m) is non existing (4%), and the parameter is not well defined to the left where the prior is slightly narrower than the posterior. The density regulation (γ) is hardly (10%) updated, and the parameter is not well defined to the right where the prior is slightly narrower than the posterior. There is hardly any (12%) updating of the maximum sustainable yield level (msyl), and the prior is slightly narrower than the posterior to the right. The updating of the catch history (c_h) is very weak (7%), and the parameter is not well defined to the left where the prior is slightly narrower than the posterior. There is a strong (66%) updating of the abundance estimate bias (β_a), and it is very clear that the posterior is narrower than the prior in both ends.

For the **inertia dynamics** (i) model in Figure 4 the updating of the population dynamic equilibrium abundance (N^*) is strong (70%), and the prior is very clearly wider than the posterior in both ends. There is hardly any (14%) updating of the exponential growth rate (r), and it is clear that the prior is narrower than the posterior to the left. The life-history parameters (p, p_0, b, a_m) are hardly (12%) updated. The yearly survival (p) has a weak (22%) updating, and the prior is slightly narrower than the posterior in both ends. There is hardly any (6%) updating of the first year survival (p_0), and the parameter is not well defined to the left where the prior is slightly narrower than the posterior. The birth rate (b) has hardly any (6%) updating, and the prior is slightly narrower than the posterior to the left. There is hardly any (14%) updating of the age of the first reproductive event (a_m); the parameter is not well defined to the right where the posterior is slightly wider than the prior. The density regulation (γ) has a weak (19%) updating; the parameter is not well defined to the left where the prior is slightly narrower than the posterior. The inertia (ι) is substantially (32%) updated, and the posterior is slightly wider than the prior to the left. There is hardly any (7%) updating of the catch history (c_h); the parameter is not well defined to the left where the posterior is slightly wider than the prior. The updating of the abundance estimate bias (β_a) is strong (72%); the parameter is strongly defined, the posterior is very clearly narrower than the prior in both ends.

Baffin Bay - Davis Strait

For the **inertia dynamics** (iB) model in Figure 5 there is a strong (66%) updating of the population dynamic equilibrium abundance (N^*); it is very clear that the prior is wider than the posterior in both ends. The updating of the exponential growth rate (r) is very weak (9%), and the posterior is slightly wider than the prior to the left. The life-history parameters (p, p_0, b, a_m) are hardly (9%) updated. The updating of the yearly survival (p) is weak (17%); the posterior is slightly wider than the prior to the right. The first year survival (p_0) has no (3%) updating; the parameter is not well defined to the left where the prior is slightly narrower than the posterior. There is hardly any (6%) updating of the birth rate (b); the parameter is not well defined to the left where the posterior is slightly wider than the prior. The age of the first reproductive event (a_m) has hardly any (9%) updating; the prior is slightly narrower than the posterior to the right. The density regulation (γ) is substantially (25%) updated, and the parameter is not well defined to the left where the posterior is slightly wider than the prior. The updating of the inertia (ι) is substantial (38%); the parameter is badly defined to the left where the posterior is wider than the prior. The catch history (c_h) is not (5%) updated; the posterior is not well updated in both ends. There is a strong (73%) updating of the abundance estimate bias (β_a); it is very clear that the posterior is narrower than the prior in both ends.

Parameter estimates

The posterior parameter estimates and their 90% credibility intervals are given in Table 7. When the posterior distributions are not well updated from the realised prior, the estimates are given basically by the priors that go into the modelling. Only parameter estimates that are based on a well updated ($w_i < 0.5$ and $u > 15\%$) posterior distribution are considered below.

Eastern Canada - West Greenland

The **short-term exponential** (e) model provides the following updated parameter estimates: a initial abundance (N_0) estimate of 1700 (90% CI:570-18000); the abundance estimate bias (β_a) is 0.015 (90% CI:0.0053-0.037).

The **density regulated growth** (d) model has the following updated parameter estimates: an estimate of the population dynamic equilibrium abundance (N^*) of 16000 (90% CI:12000-25000); the estimate of the abundance estimate bias (β_a) is 0.0029 (90% CI:0.0014-0.016).

The **inertia dynamics** (i) model provides the following updated parameter estimates: the population dynamic equilibrium abundance (N^*) is 30000 (90% CI:24000-35000); the estimate of the abundance estimate bias (β_a) is 0.0088 (90% CI:0.0032-0.023).

Baffin Bay - Davis Strait

The **inertia dynamics** (iB) model has the following updated parameter estimates: the population dynamic equilibrium abundance (N^*) was estimated to 34000 (90% CI:23000-40000); the estimate of the abundance estimate bias (β_a) is 0.0069 (90% CI:0.0026-0.018).

Population dynamics

The estimated population dynamic trajectories are shown in Figure 6.

Eastern Canada - West Greenland

Relating to long-term dynamics, the **inertia dynamics** (i) model is substantially ($K = 9.81$) supported over the **density regulated growth** (d) model by Bayes factor. The **inertia dynamics** (i) model describes the long-term population dynamics from 1719 to 2020. It is estimated that the abundance has declined from a population dynamic equilibrium of 30000 (90% CI:24000-35000) individuals in 1719 to a minumum in 1888 with 1700 (90% CI:510-4900) individuals. It can be expected that the population will increase to a projected abundance estimate of 10000 (90% CI:5200-20000) in 2020 (assuming an average post 2010 catch of 5 per year). The depletion ratio in 2011 is estimated to 0.29 (90% CI:0.15-0.58), with an abundance of 8700 (90% CI:4400-16000) and a yearly replacement of 150 (90% CI:52-450) individuals. From 2006 to 2011 the population had on average increased by 142 individuals per year.

The **short-term exponential** (e) model describes the short-term population dynamics from 1970 to 2020. Population increase from an initial abundance in 1970 with 1700 (90% CI:570-18000) individuals to a projected estimate with 12000 (90% CI:3400-27000) individuals in 2020 (assuming a post 2010 catch of 5 per year). The abundance in 2011 is estimated to 8500 (90% CI:3900-17000) with a yearly replacement of 320 (90% CI:−130-910) individuals. From 2006 to 2011 the population had on average increased by 249 individuals per year.

Baffin Bay - Davis Strait

The **inertia dynamics** (iB) model describes the long-term population dynamics from 1719 to 2020. The abundance has declined from a population dynamic equilibrium in 1719 with 34000 (90% CI:23000-40000) individuals to a minumum of 3400 (90% CI:590-8500) individuals in 1888. It can be expected that the population increase to a projected abundance estimate of 9100 (90% CI:4500-18000) individuals in 2020 (assuming an average post 2010 catch of 3 per year). For 2011 it is estimated that the depletion ratio is 0.25 (90% CI:0.13-0.52), that the abundance is 8200 (90% CI:4200-16000) individuals, and that the yearly replacement is 70 (90% CI:14-330). From 2006 to 2011 the population had on average increased by 95 individuals per year.

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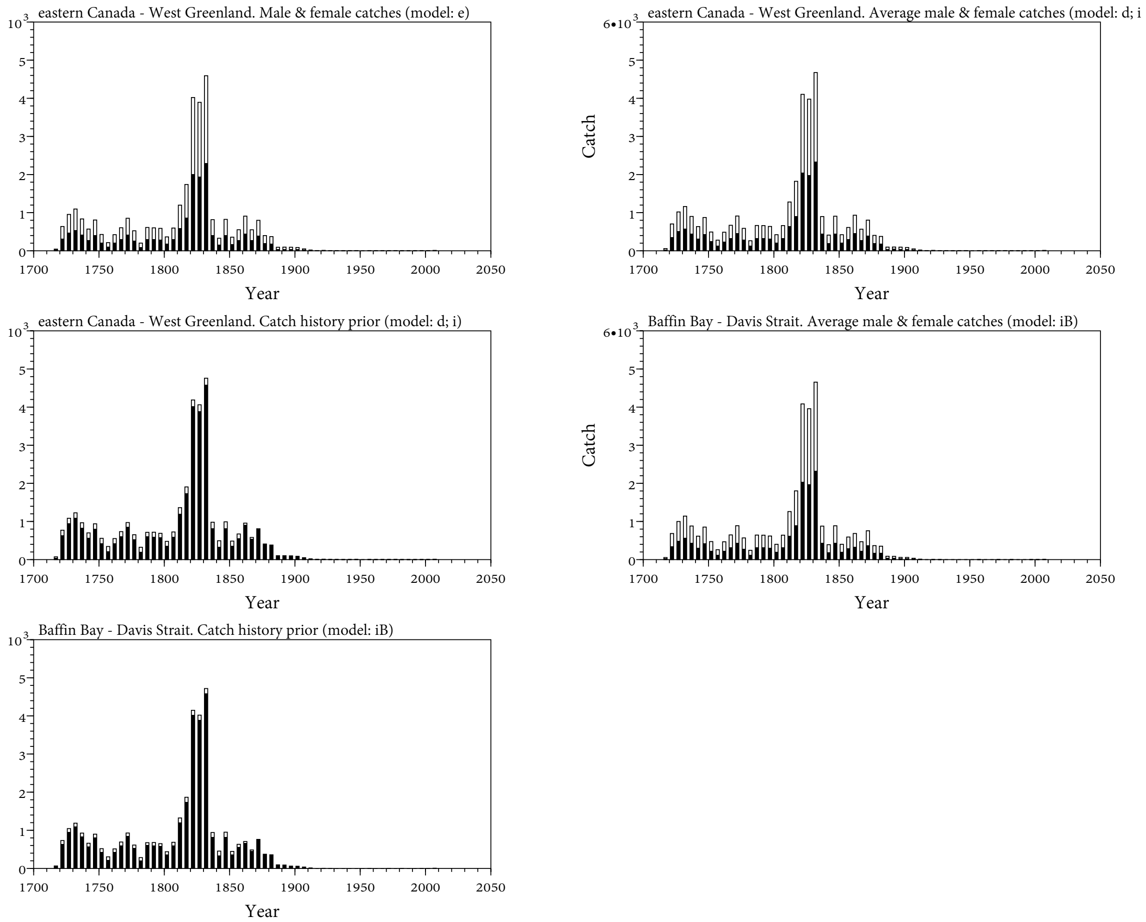

Figure 1: The historical catches of males (solid bars) and females (open bars), and prior range in total catches (minimum by solid bars; maximum by open bars). Data from .

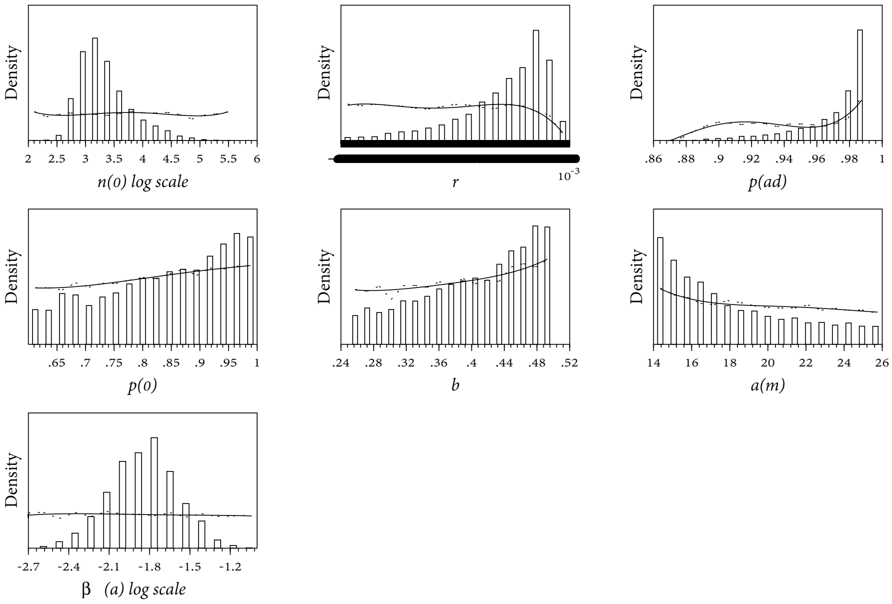

Figure 2: Realised prior (curve) and posterior (bars) distributions for model e.

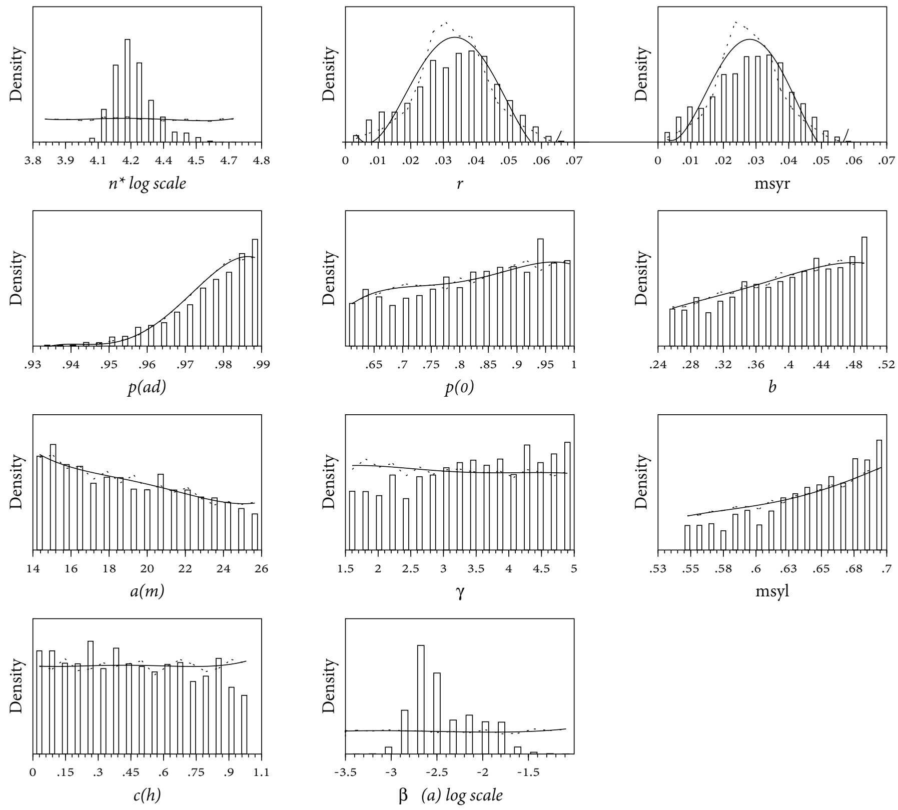

Figure 3: Realised prior (curve) and posterior (bars) distributions for model d.

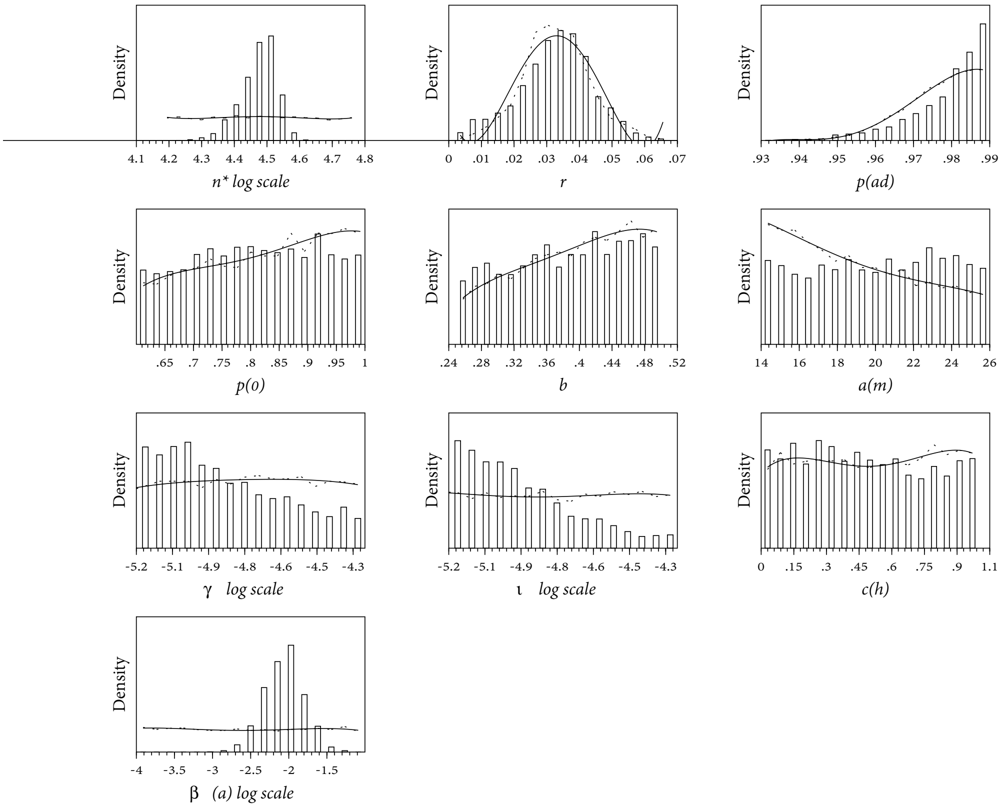

Figure 4: Realised prior (curve) and posterior (bars) distributions for model i.

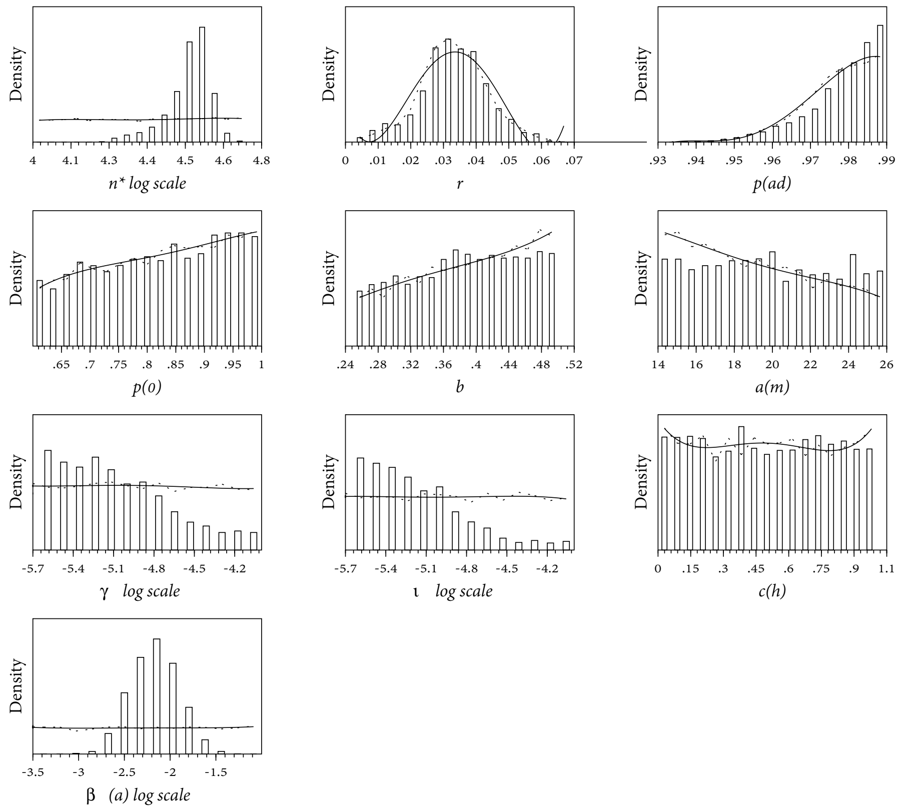

Figure 5: Realised prior (curve) and posterior (bars) distributions for model iB.

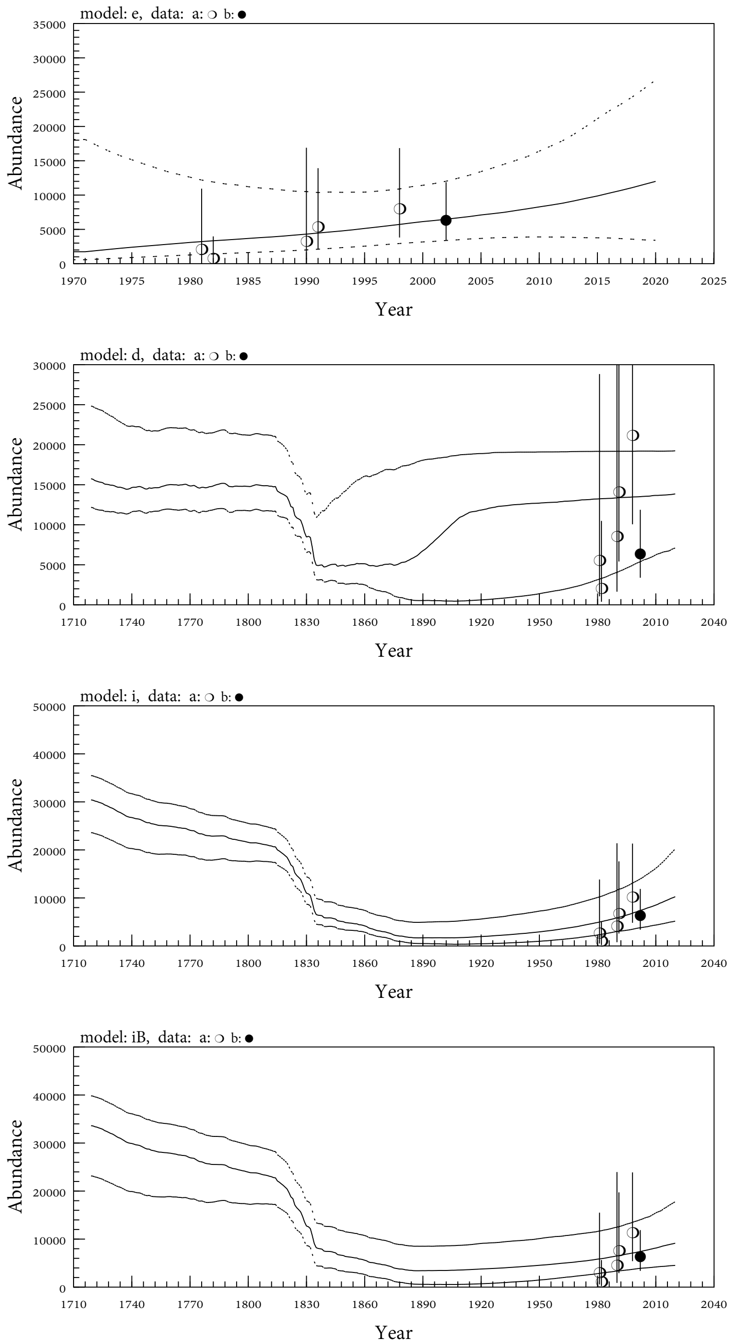

Figure 6: The projected median and 90% credibility interval of the different models.

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M	e	d	i	iB
$\tilde{p}_0$	-	-	-	-
$\tilde{p}_1$	1|1	1|1	1|1	1|1
$\tilde{p}_2$	1|1	1|1	1|1	1|1
$\tilde{p}_3$	1|1	1|1	1|1	1|1
$\tilde{p}_4$	1|1	1|1	1|1	1|1
$\tilde{p}_5$	1|1	1|1	1|1	1|1
$\tilde{p}_6$	1|1	1|1	1|1	1|1
$\tilde{p}_7$	1|1	1|1	1|1	1|1
$\tilde{p}_8$	1|1	1|1	1|1	1|1
$\tilde{p}_9$	1|1	1|1	1|1	1|1
$\tilde{p}_{10}$	1|1	1|1	1|1	1|1
$\tilde{p}_{11}$	1|1	1|1	1|1	1|1
$\tilde{p}_{12}$	1|1	1|1	1|1	1|1
$\tilde{p}_{13}$	1|1	1|1	1|1	1|1
$\tilde{p}_{14}$	1|1	1|1	1|1	1|1
$\tilde{p}_{15}$	1|1	1|1	1|1	1|1
$\tilde{p}_{16}$	1|1	1|1	1|1	1|1
$\tilde{p}_{17}$	1|1	1|1	1|1	1|1
$\tilde{p}_{18}$	1|1	1|1	1|1	1|1
$\tilde{p}_{19}$	1|1	1|1	1|1	1|1
$\tilde{p}_{20}$	1|1	1|1	1|1	1|1
$\tilde{p}_{21}$	1|1	1|1	1|1	1|1
$\tilde{p}_{22}$	1|1	1|1	1|1	1|1
$\tilde{p}_{23}$	1|1	1|1	1|1	1|1
$\tilde{p}_{24}$	1|1	1|1	1|1	1|1
$\tilde{p}_{25}$	1|1	1|1	1|1	1|1
$\tilde{p}_{26}$	1|1	1|1	1|1	1|1
$\tilde{p}_{27+}$	1|1	1|1	1|1	1|1

Table 2: **Age-structured relative survival.** The relative survival $\tilde{p}_a$ of age-class a, is given $(m|f)$ for males (m) and females (f) seperatedly. Models (M) are indicated by symbols, and - indicate that $\tilde{p}_0 \leq 1$ is a prior with $p_0 = \tilde{p}_0 p_1$.

M	e	d	i	iB
$\tilde{c}_0$	0|0	0|0	0|0	0|0
$\tilde{c}_1$	.037|.037	.037|.037	.037|.037	.037|.037
$\tilde{c}_2$	.037|.037	.037|.037	.037|.037	.037|.037
$\tilde{c}_3$	.037|.037	.037|.037	.037|.037	.037|.037
$\tilde{c}_4$	.037|.037	.037|.037	.037|.037	.037|.037
$\tilde{c}_5$	.037|.037	.037|.037	.037|.037	.037|.037
$\tilde{c}_6$	.037|.037	.037|.037	.037|.037	.037|.037
$\tilde{c}_7$	.037|.037	.037|.037	.037|.037	.037|.037
$\tilde{c}_8$	.037|.037	.037|.037	.037|.037	.037|.037
$\tilde{c}_9$	.037|.037	.037|.037	.037|.037	.037|.037
$\tilde{c}_{10}$	.037|.037	.037|.037	.037|.037	.037|.037
$\tilde{c}_{11}$	.037|.037	.037|.037	.037|.037	.037|.037
$\tilde{c}_{12}$	.037|.037	.037|.037	.037|.037	.037|.037
$\tilde{c}_{13}$	.037|.037	.037|.037	.037|.037	.037|.037
$\tilde{c}_{14}$	.037|.037	.037|.037	.037|.037	.037|.037
$\tilde{c}_{15}$	.037|.037	.037|.037	.037|.037	.037|.037
$\tilde{c}_{16}$	.037|.037	.037|.037	.037|.037	.037|.037
$\tilde{c}_{17}$	.037|.037	.037|.037	.037|.037	.037|.037
$\tilde{c}_{18}$	.037|.037	.037|.037	.037|.037	.037|.037
$\tilde{c}_{19}$	.037|.037	.037|.037	.037|.037	.037|.037
$\tilde{c}_{20}$	.037|.037	.037|.037	.037|.037	.037|.037
$\tilde{c}_{21}$	.037|.037	.037|.037	.037|.037	.037|.037
$\tilde{c}_{22}$	.037|.037	.037|.037	.037|.037	.037|.037
$\tilde{c}_{23}$	.037|.037	.037|.037	.037|.037	.037|.037
$\tilde{c}_{24}$	.037|.037	.037|.037	.037|.037	.037|.037
$\tilde{c}_{25}$	.037|.037	.037|.037	.037|.037	.037|.037
$\tilde{c}_{26}$	.037|.037	.037|.037	.037|.037	.037|.037
$\tilde{c}_{27+}$	.037|.037	.037|.037	.037|.037	.037|.037

Table 3: **Age-structured catch selectivity.** The catch selectivity $\tilde{c}_a$ for individuals in age-class a, is given $(m|f)$ for males (m) and females (f) seperately. Models (M) are indicated by symbols.

M	e	d	i	iB
$\tilde{b}_m$	1	1	1	1
$\tilde{b}_{m+1}$	1	1	1	1
$\tilde{b}_{m+2}$	1	1	1	1
$\tilde{b}_{m+3}$	1	1	1	1
$\tilde{b}_{m+4}$	1	1	1	1
$\tilde{b}_{m+5}$	1	1	1	1
$\tilde{b}_{m+6}$	1	1	1	1
$\tilde{b}_{m+7}$	1	1	1	1
$\tilde{b}_{m+8}$	1	1	1	1
$\tilde{b}_{m+9}$	1	1	1	1
$\tilde{b}_{m+10}$	1	1	1	1
$\tilde{b}_{m+11}$	1	1	1	1
$\tilde{b}_{m+12}$	1	1	1	1
$\tilde{b}_{m+13}$	1	1	1	1
$\tilde{b}_{m+14^+}$	1	1	1	1

Table 4: **Age-structured relative birth rate.** The relative birth rate $\tilde{b}_a$ for females in age-class a. Models (M) are indicated by symbols.

M	N_0	N^*	r	p_0	b	a_m	ϑ
e	$.1,400^U$	-	$-0.07,.07^u$	$.6,1^u$	$.25,.5^u$	$14,26^u$	.5
d	-	$6,50^U$	$15,420^b$	$.6,1^u$	$.25,.5^u$	$14,26^u$	.5
i	-	$15,60^U$	$15,420^b$	$.6,1^u$	$.25,.5^u$	$14,26^u$	.5
iB	-	$10,50^U$	$15,420^b$	$.6,1^u$	$.25,.5^u$	$14,26^u$	.5

M	γ	ι	c_h	β_a
e	-	-	-	$.001,.1^U$
d	$1.5,5^u$	-	$0,1^u$	$.0001,.1^U$
i	$5e\text{-}6,5e\text{-}5^U$	$5e\text{-}6,5e\text{-}5^U$	$0,1^u$	$.0001,.1^U$
iB	$1e\text{-}6,.0001^U$	$1e\text{-}6,.0001^U$	$0,1^u$	$.0001,.1^U$

Table 5: **Prior distributions** for the different models (M). The list of parameters: N_0 is the initial abundance, N^* the population dynamic equilibrium abundance, r the exponential growth rate, p_0 the first year survival, b the birth rate, a_m the age of the first reproductive event, ϑ the female fraction at birth, γ the density regulation, ι the inertia, c_h the catch history, and β_i the abundance estimate bias (i: data reference). Abundance is given in thousands. The prior probability distribution is given by superscripts; p: fixed value, u: uniform (min,max), U: log uniform (min,max), and b: beta (a,b).

M	n_S	n_R	Weight	Unique	Max
e	300	5	4.978	3469	8
d	500	5	2.025	3038	24
i	1000	5	2.535	2222	17
iB	1000	5	2.507	3088	10

Table 6: **Sampling statistics** for the different models (M). The number of parameter sets in the sample (n_S) and the resample (n_R), the maximum importance weight of a draw relative to the total importance weight of all draws, the number of unique parameter sets in the resample, and the maximum number of occurrences of a unique parameter set in the resample. n_S and n_R are given in thousands.

M		N_0	N^*	r	msyr	p	p_0	b	a_m	γ	ι
	$x_{.5}$	1.7	-	.038	-	.98	.86	.42	17	-	-
	$x_{.05}$	.57	-	-0.027	-	.92	.64	.27	14	-	-
$^+$e	$x_{.95}$	18	-	.061	-	.99	.99	.49	25	-	-
	$x_{.5}$	-	16	.034	.029	.98	.84	.41	19	3.5	-
	$x_{.05}$	-	12	.01	.0085	.96	.63	.27	14	1.7	-
$^+$d	$x_{.95}$	-	25	.054	.047	.99	.98	.49	25	4.9	-
	$x_{.5}$	-	30	.033	-	.98	.81	.39	20	1.1e-5	9.2e-6
	$x_{.05}$	-	24	.011	-	.96	.63	.27	15	5.5e-6	5.3e-6
$^+$i	$x_{.95}$	-	35	.051	-	.99	.98	.49	25	4e-5	3.1e-5
	$x_{.5}$	-	34	.033	-	.98	.83	.39	20	4.5e-6	3.4e-6
	$x_{.05}$	-	23	.013	-	.96	.63	.27	15	1.2e-6	1.1e-6
$^+$iB	$x_{.95}$	-	40	.05	-	.99	.98	.49	25	4.4e-5	2.6e-5

M	msyl	c_h	N_t	d_t	β_a
	-	-	8.5	-	.015
	-	-	3.9	-	.0053
$^+$e	-	-	17	-	.037
	.65	.45	14	1	.0029
	.56	.041	6.4	.29	.0014
$^+$d	.7	.93	19	1	.016
	-	.46	8.7	.29	.0088
	-	.041	4.4	.15	.0032
$^+$i	-	.95	16	.58	.023
	-	.49	8.2	.25	.0069
	-	.046	4.2	.13	.0026
$^+$iB	-	.95	16	.52	.018

Table 7: **Parameter estimates** for the different models (M). Estimates are given by the median ($x_{.5}$) and the 90% credibility interval ($x_{.05}$ - $x_{.95}$) of the postreior distributions. Abundance is given in thousands. The selected models are indicated a superscript +.

On population dynamics of eastern Canada - West Greenland bowhead whales

Lars Witting

This supplement to IWC/SC/63/AWMP3 - Revised gives correlation matrixes for the parameters in the different models, and it also lists the applied catch histories.

Par	N_0	r	p	p_0	b	a_m	N_t	β_a
N_0	1	-	-	-	-	-	-	-
r	-0.71	1	-	-	-	-	-	-
p	-0.72	0.94	1	-	-	-	-	-
p_0	-0.15	0.31	0.15	1	-	-	-	-
b	-0.16	0.38	0.17	0.08	1	-	-	-
a_m	0.33	-0.55	-0.36	-0.14	-0.18	1	-	-
N_t	-0.18	0.46	0.42	0.16	0.2	-0.25	1	-
β_a	-0.31	0.38	0.35	0.22	0.26	-0.091	-0.34	1

Table 1: **Parameter correlation matrix** for model e.

Par	N^*	r	msyr	p	p_0	b	a_m	γ	msyl	c_h	N_t	d_t	β_a
N^*	1	-	-	-	-	-	-	-	-	-	-	-	-
r	-0.75	1	-	-	-	-	-	-	-	-	-	-	-
msyr	-0.76	0.99	1	-	-	-	-	-	-	-	-	-	-
p	-0.68	0.76	0.76	1	-	-	-	-	-	-	-	-	-
p_0	-0.19	0.29	0.29	0	1	-	-	-	-	-	-	-	-
b	-0.2	0.35	0.36	-0.067	-0.0088	1	-	-	-	-	-	-	-
a_m	0.02	-0.24	-0.22	0.2	0.06	-0.039	1	-	-	-	-	-	-
γ	-0.23	0.08	0.21	0.07	0.01	0.04	-0.0049	1	-	-	-	-	-
msyl	-0.23	0.08	0.22	0.07	0.01	0.04	-0.0027	0.99	1	-	-	-	-
c_h	0.12	-0.021	-0.023	-0.021	0.01	0.01	0.03	-0.024	-0.026	1	-	-	-
N_t	-0.079	0.38	0.37	0.38	0.04	0.08	0	0	0	0.03	1	-	-
d_t	-0.61	0.76	0.76	0.69	0.17	0.23	-0.035	0.17	0.17	-0.038	0.8	1	-
β_a	0.29	-0.51	-0.51	-0.46	-0.058	-0.11	0.11	-0.073	-0.072	0.01	-0.72	-0.79	1

Table 2: **Parameter correlation matrix** for model d.

Par	N^*	r	p	p_0	b	a_m	γ	ι	c_h	N_t	d_t	β_a
N^*	1	-	-	-	-	-	-	-	-	-	-	-
r	0.39	1	-	-	-	-	-	-	-	-	-	-
p	0.57	0.74	1	-	-	-	-	-	-	-	-	-
p_0	-0.049	0.2	-0.084	1	-	-	-	-	-	-	-	-
b	-0.051	0.32	-0.11	-0.031	1	-	-	-	-	-	-	-
a_m	0.17	-0.25	0.18	0.07	0.01	1	-	-	-	-	-	-
γ	-0.68	-0.14	-0.14	0	-0.0006	0.04	1	-	-	-	-	-
ι	-0.5	-0.32	-0.31	-0.018	-0.067	-0.016	0.12	1	-	-	-	-
c_h	0.16	-0.094	-0.079	-0.042	-0.012	-0.0024	0.02	0.01	1	-	-	-
N_t	0.04	0.08	0.03	0.05	0.02	-0.058	0.03	0.04	0.01	1	-	-
d_t	-0.26	-0.029	-0.14	0.07	0.04	-0.11	0.24	0.2	-0.029	0.94	1	-
β_a	-0.33	-0.18	-0.24	0.05	0.11	0.05	0.11	0.22	0.04	-0.51	-0.4	1

Table 3: **Parameter correlation matrix** for model i.

Par	N^*	r	p	p_0	b	a_m	γ	ι	c_h	N_t	d_t	β_a
N^*	1	-	-	-	-	-	-	-	-	-	-	-
r	0.31	1	-	-	-	-	-	-	-	-	-	-
p	0.36	0.69	1	-	-	-	-	-	-	-	-	-
p_0	0	0.23	-0.11	1	-	-	-	-	-	-	-	-
b	-0.03	0.26	-0.2	-0.06	1	-	-	-	-	-	-	-
a_m	0.04	-0.28	0.21	0.01	0.03	1	-	-	-	-	-	-
γ	-0.72	-0.15	-0.11	-0.038	-0.027	0.03	1	-	-	-	-	-
ι	-0.46	-0.17	-0.13	-0.014	-0.034	0.03	0.13	1	-	-	-	-
c_h	0.09	0.01	-0.019	0.04	-0.016	-0.045	0.02	-0.01	1	-	-	-
N_t	0.04	0.01	-0.017	0.02	0	-0.05	0.11	0.09	-0.0048	1	-	-
d_t	-0.38	-0.1	-0.15	0.02	0.01	-0.066	0.42	0.29	-0.032	0.89	1	-
β_a	-0.43	-0.27	-0.3	0.06	0.07	0.09	0.13	0.29	0.03	-0.45	-0.25	1

Table 4: **Parameter correlation matrix** for model iB.

Year	C	Year	C	Year	C	Year	C	Year	C	Year	C	Year	C
1970	0	1978	0	1986	0	1994	1	2002	0	2010	5	2018	-
1971	2	1979	1	1987	0	1995	0	2003	1	2011	-	2019	-
1972	0	1980	1	1988	0	1996	1	2004	1	2012	-	2020	-
1973	1	1981	0	1989	0	1997	0	2005	1	2013	-		
1974	0	1982	0	1990	0	1998	1	2006	0	2014	-		
1975	3	1983	0	1991	0	1999	0	2007	0	2015	-		
1976	0	1984	0	1992	0	2000	1	2008	3	2016	-		
1977	0	1985	1	1993	0	2001	0	2009	6	2017	-		

Table 5: **Low catch history; eastern Canada - West Greenland.** Low catch history. Used in models e, d (lower limit on catch prior), and i (lower limit on catch prior).

Year	C	Year	C	Year	C	Year	C	Year	C	Year	C	Year	C
1719	72	1763	173	1807	144	1851	98	1895	9	1939	1	1983	0
1720	180	1764	61	1808	119	1852	77	1896	18	1940	2	1984	0
1721	102	1765	121	1809	149	1853	112	1897	16	1941	1	1985	1
1722	174	1766	60	1810	191	1854	117	1898	17	1942	0	1986	0
1723	145	1767	112	1811	131	1855	64	1899	38	1943	0	1987	0
1724	170	1768	248	1812	239	1856	235	1900	25	1944	0	1988	0
1725	305	1769	193	1813	81	1857	70	1901	22	1945	3	1989	0
1726	163	1770	123	1814	721	1858	125	1902	12	1946	1	1990	0
1727	220	1771	66	1815	300	1859	178	1903	16	1947	1	1991	0
1728	239	1772	284	1816	391	1860	168	1904	12	1948	0	1992	0
1729	157	1773	287	1817	365	1861	307	1905	32	1949	0	1993	0
1730	249	1774	212	1818	423	1862	179	1906	7	1950	0	1994	1
1731	289	1775	52	1819	427	1863	118	1907	3	1951	0	1995	0
1732	261	1776	199	1820	804	1864	185	1908	5	1952	0	1996	1
1733	164	1777	220	1821	963	1865	181	1909	3	1953	0	1997	0
1734	263	1778	108	1822	392	1866	121	1910	13	1954	0	1998	1
1735	260	1779	73	1823	1399	1867	76	1911	6	1955	1	1999	0
1736	299	1780	136	1824	629	1868	152	1912	1	1956	1	2000	1
1737	186	1781	59	1825	459	1869	51	1913	0	1957	0	2001	0
1738	142	1782	39	1826	490	1870	106	1914	0	1958	0	2002	0
1739	79	1783	43	1827	991	1871	168	1915	0	1959	1	2003	1
1740	141	1784	47	1828	1220	1872	125	1916	0	1960	0	2004	1
1741	165	1785	29	1829	900	1873	183	1917	0	1961	1	2005	1
1742	79	1786	58	1830	193	1874	227	1918	1	1962	0	2006	0
1743	105	1787	99	1831	443	1875	121	1919	5	1963	0	2007	0
1744	210	1788	229	1832	1516	1876	82	1920	1	1964	1	2008	3
1745	238	1789	300	1833	1701	1877	97	1921	3	1965	1	2009	6
1746	249	1790	142	1834	905	1878	18	1922	5	1966	0	2010	5
1747	167	1791	194	1835	199	1879	91	1923	2	1967	1	2011	-
1748	33	1792	68	1836	95	1880	126	1924	0	1968	0	2012	-
1749	251	1793	154	1837	122	1881	58	1925	1	1969	0	2013	-
1750	90	1794	161	1838	448	1882	89	1926	1	1970	0	2014	-
1751	136	1795	123	1839	117	1883	25	1927	0	1971	2	2015	-
1752	154	1796	141	1840	48	1884	84	1928	1	1972	0	2016	-
1753	133	1797	110	1841	48	1885	33	1929	1	1973	1	2017	-
1754	45	1798	166	1842	88	1886	24	1930	3	1974	0	2018	-
1755	77	1799	149	1843	168	1887	22	1931	0	1975	3	2019	-
1756	67	1800	158	1844	142	1888	10	1932	0	1976	0	2020	-
1757	41	1801	63	1845	406	1889	13	1933	0	1977	0		
1758	93	1802	66	1846	127	1890	24	1934	0	1978	0		
1759	66	1803	62	1847	113	1891	8	1935	0	1979	1		
1760	116	1804	129	1848	101	1892	13	1936	0	1980	1		
1761	105	1805	140	1849	243	1893	34	1937	0	1981	0		
1762	97	1806	173	1850	82	1894	23	1938	0	1982	0		

Table 6: **High catch history; eastern Canada - West Greenland.** High catch history. Used in models d (upper limit on catch prior) and i (upper limit in catch prior).

Year	C	Year	C	Year	C	Year	C	Year	C	Year	C	Year	C
1719	44	1763	147	1807	125	1851	74	1895	5	1939	0	1983	0
1720	152	1764	35	1808	86	1852	53	1896	6	1940	0	1984	0
1721	74	1765	95	1809	116	1853	88	1897	10	1941	0	1985	0
1722	148	1766	34	1810	158	1854	93	1898	6	1942	0	1986	0
1723	119	1767	85	1811	98	1855	40	1899	28	1943	0	1987	0
1724	144	1768	221	1812	206	1856	211	1900	17	1944	0	1988	0
1725	279	1769	167	1813	47	1857	46	1901	10	1945	0	1989	0
1726	137	1770	97	1814	688	1858	101	1902	11	1946	0	1990	0
1727	194	1771	39	1815	267	1859	154	1903	9	1947	0	1991	0
1728	213	1772	259	1816	357	1860	132	1904	9	1948	0	1992	0
1729	131	1773	267	1817	332	1861	268	1905	20	1949	0	1993	0
1730	223	1774	187	1818	389	1862	139	1906	6	1950	0	1994	0
1731	263	1775	26	1819	394	1863	35	1907	3	1951	0	1995	0
1732	235	1776	172	1820	770	1864	91	1908	4	1952	0	1996	0
1733	138	1777	199	1821	929	1865	129	1909	2	1953	0	1997	0
1734	237	1778	82	1822	359	1866	90	1910	11	1954	0	1998	1
1735	234	1779	46	1823	1366	1867	57	1911	1	1955	0	1999	0
1736	273	1780	113	1824	596	1868	142	1912	0	1956	1	2000	0
1737	160	1781	27	1825	425	1869	36	1913	0	1957	0	2001	0
1738	116	1782	18	1826	457	1870	96	1914	0	1958	0	2002	0
1739	53	1783	21	1827	958	1871	150	1915	0	1959	0	2003	0
1740	115	1784	23	1828	1187	1872	115	1916	0	1960	0	2004	1
1741	139	1785	7	1829	867	1873	171	1917	0	1961	0	2005	0
1742	53	1786	39	1830	160	1874	219	1918	0	1962	0	2006	0
1743	79	1787	80	1831	410	1875	104	1919	0	1963	0	2007	0
1744	184	1788	210	1832	1483	1876	78	1920	0	1964	0	2008	1
1745	212	1789	276	1833	1668	1877	91	1921	1	1965	0	2009	4
1746	223	1790	119	1834	871	1878	12	1922	0	1966	0	2010	3
1747	141	1791	175	1835	166	1879	77	1923	1	1967	0	2011	-
1748	7	1792	49	1836	62	1880	120	1924	0	1968	0	2012	-
1749	225	1793	120	1837	89	1881	51	1925	0	1969	0	2013	-
1750	64	1794	139	1838	415	1882	79	1926	0	1970	0	2014	-
1751	110	1795	102	1839	84	1883	18	1927	0	1971	0	2015	-
1752	128	1796	121	1840	15	1884	79	1928	1	1972	0	2016	-
1753	107	1797	89	1841	15	1885	29	1929	0	1973	1	2017	-
1754	19	1798	147	1842	55	1886	17	1930	0	1974	0	2018	-
1755	51	1799	129	1843	135	1887	18	1931	0	1975	0	2019	-
1756	41	1800	133	1844	109	1888	8	1932	0	1976	0	2020	-
1757	15	1801	40	1845	373	1889	11	1933	0	1977	0		
1758	67	1802	44	1846	94	1890	22	1934	0	1978	0		
1759	40	1803	38	1847	80	1891	7	1935	0	1979	0		
1760	90	1804	110	1848	67	1892	7	1936	0	1980	1		
1761	79	1805	119	1849	210	1893	30	1937	0	1981	0		
1762	71	1806	151	1850	49	1894	16	1938	0	1982	0		

Table 7: **Low catch history; Baffin Bay - Davis Strait.** Low catch history. Used in model iB (lower limit on catch prior).

Year	C	Year	C	Year	C	Year	C	Year	C	Year	C	Year	C
1719	64	1763	165	1807	136	1851	90	1895	6	1939	0	1983	0
1720	172	1764	53	1808	111	1852	69	1896	7	1940	0	1984	0
1721	94	1765	113	1809	141	1853	104	1897	11	1941	0	1985	0
1722	166	1766	52	1810	183	1854	109	1898	7	1942	0	1986	0
1723	137	1767	103	1811	123	1855	56	1899	29	1943	0	1987	0
1724	162	1768	239	1812	231	1856	227	1900	18	1944	0	1988	0
1725	297	1769	185	1813	73	1857	62	1901	10	1945	0	1989	0
1726	155	1770	115	1814	713	1858	117	1902	11	1946	0	1990	0
1727	212	1771	55	1815	292	1859	170	1903	9	1947	0	1991	0
1728	231	1772	275	1816	383	1860	148	1904	9	1948	0	1992	0
1729	149	1773	279	1817	357	1861	274	1905	20	1949	0	1993	0
1730	241	1774	204	1818	415	1862	145	1906	6	1950	0	1994	0
1731	281	1775	44	1819	419	1863	41	1907	3	1951	0	1995	0
1732	253	1776	191	1820	796	1864	97	1908	4	1952	0	1996	0
1733	156	1777	212	1821	955	1865	135	1909	2	1953	0	1997	0
1734	255	1778	100	1822	384	1866	96	1910	11	1954	0	1998	1
1735	252	1779	65	1823	1391	1867	63	1911	1	1955	0	1999	0
1736	291	1780	128	1824	621	1868	148	1912	0	1956	1	2000	0
1737	178	1781	51	1825	451	1869	42	1913	0	1957	0	2001	0
1738	134	1782	31	1826	482	1870	98	1914	0	1958	0	2002	0
1739	71	1783	35	1827	983	1871	152	1915	0	1959	0	2003	0
1740	133	1784	39	1828	1212	1872	117	1916	0	1960	0	2004	1
1741	157	1785	21	1829	892	1873	173	1917	0	1961	0	2005	0
1742	71	1786	50	1830	185	1874	221	1918	0	1962	0	2006	0
1743	97	1787	91	1831	435	1875	106	1919	0	1963	0	2007	0
1744	202	1788	221	1832	1508	1876	80	1920	0	1964	0	2008	1
1745	230	1789	292	1833	1693	1877	93	1921	1	1965	0	2009	4
1746	241	1790	134	1834	897	1878	14	1922	0	1966	0	2010	3
1747	159	1791	186	1835	191	1879	79	1923	1	1967	0	2011	-
1748	25	1792	60	1836	87	1880	122	1924	0	1968	0	2012	-
1749	243	1793	146	1837	114	1881	53	1925	0	1969	0	2013	-
1750	82	1794	153	1838	440	1882	81	1926	0	1970	0	2014	-
1751	128	1795	115	1839	109	1883	20	1927	0	1971	0	2015	-
1752	146	1796	133	1840	40	1884	81	1928	1	1972	0	2016	-
1753	125	1797	102	1841	40	1885	31	1929	0	1973	1	2017	-
1754	37	1798	158	1842	80	1886	19	1930	0	1974	0	2018	-
1755	69	1799	141	1843	160	1887	20	1931	0	1975	0	2019	-
1756	59	1800	150	1844	134	1888	10	1932	0	1976	0	2020	-
1757	33	1801	55	1845	398	1889	13	1933	0	1977	0		
1758	85	1802	58	1846	119	1890	24	1934	0	1978	0		
1759	58	1803	54	1847	105	1891	8	1935	0	1979	0		
1760	108	1804	121	1848	93	1892	8	1936	0	1980	1		
1761	97	1805	132	1849	235	1893	31	1937	0	1981	0		
1762	89	1806	165	1850	74	1894	17	1938	0	1982	0		

Table 8: **High catch history; Baffin Bay - Davis Strait.** High catch history. Used in model iB (upper limit on catch prior).